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THE MAXIMUM PRINCIPLE FOR COOPERATIVE WEAKLY COUPLED ELLIPTIC SYSTEMS AND SOME APPLICATIONS

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Abstract. In this work we obtain some characterizations of the maximum principle for weakly coupled elliptic cooperative systems. Roughly speaking, it is shown that a cooperative system behaves as one single equation. This fact is used to analyze the existence and uniqueness of positive principal eigenvalues for some boundary value problems associated with the system. The theory developed here provides us with a systematic analysis procedure sharpening most of the results in the references. This establishment of connections and relationships between the different problems treated separately in the literature is our main goal.

1. Introduction. Let Ω be a bounded domain of $\mathbf{R}^N$ with regular enough boundary, say $\partial\Omega$, and consider the following set of second order uniformly elliptic operators

$$L_k(D) = -\sum_{i,j=1}^{N} \alpha_{ij}^k(x) D_i D_j + \sum_{i=1}^{N} \alpha_i^k(x) D_i\,, \quad k = 1, \dots, n\,, \tag{1.1}$$

with all the coefficients α_{ij}^k, $\alpha_i^k, i, j \in \{1, \dots, N\}$, $k = 1, \dots, n$, in the Banach space of Hölder continuous functions $Y := C^\nu(\overline{\Omega}; \mathbf{R})$, for some $\nu > 0$. Let $A(x) = (a_{ij}(x))_{i,j=1,\dots,n}$ be a cooperative $n \times n$ matrix such that $a_{ij} \in Y$ and $a_{ij}(x) > 0$ for every $i, j \in \{1, \dots, n\}$, $i \neq j$, $x \in \Omega$.

In Section 2 we shall characterize whether the following boundary value problem satisfies the strong maximum principle

$$\begin{cases} L_k(D) u_k = \sum_{j=1}^n a_{kj}(x) u_j + f_k(x) & \text{in } \Omega\,, \\ u_k = 0 \quad \text{on } \partial\Omega\,, \end{cases} \qquad k = 1, \dots, n\,. \tag{1.2}$$

In (1.2) all the f_j are given functions, $f_j \in Y$. Under these assumptions it is well known that any solution $\vec{u} := (u_1, \dots, u_n)$ of (1.2) lies in $U := X^n$, being

$$X := \{u \in C^{2+\nu}(\overline{\Omega}; \mathbf{R})\,, \quad u = 0 \quad \text{on } \partial\Omega\}\,,$$

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