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VISCOSITY SOLUTIONS OF MONOTONE SYSTEMS FOR DIRICHLET PROBLEMS

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Abstract. We prove the existence of viscosity solutions for monotone elliptic systems of 2nd order fully nonlinear PDEs with generalized Dirichlet boundary condition. For such systems, a comparison principle is not always available.

1. Introduction. In this paper we consider the following system of second-order fully nonlinear PDEs:

$$F_k(x, u(x), Du_k(x), D^2u_k(x)) = 0 \quad \text{for} \quad x \in \Omega, \ k \in A \equiv \{1, 2, \dots, m\}, \tag{1.1}$$

where Ω is a bounded open set in $\mathbb{R}^n$, $F = (F_1, \dots, F_m) : \bar{\Omega} \times \mathbb{R}^m \times \mathbb{R}^n \times \mathbb{S}^n \to \mathbb{R}^m$ is given and $u = (u_1, \dots, u_m) : \bar{\Omega} \to \mathbb{R}^m$ is an unknown function. Here $\mathbb{S}^n$ denotes the space of symmetric matrices of order n.

Throughout this paper we will assume that F is monotone in the sense of Ishii [4]; see also Ishii-Koike [6]. We note that our monotone condition holds not only for a switching game and weakly coupled systems but also for a system which has no comparison principle. For these examples we refer to [6] and [4].

In [6] we obtained the existence and uniqueness of continuous viscosity solutions of (1.1) under suitable hypotheses. We remark that the proof of the existence of continuous viscosity solutions in [6] was done via Schauder's fixed point theorem. Recently, Ishii in [4] has shown that Perron's method gives an existence result for the monotone system.

On the other hand, in order to study boundary value problems, Ishii in [5] pointed out that we should treat not only the equation but also the boundary condition in the viscosity sense, especially for degenerate elliptic PDEs. For the details we only refer to "User's Guide" [2] and its references. For the monotone system, recently, Koike in [9] has obtained the uniqueness of continuous viscosity solutions

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