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BLOW-UP FOR SOLUTIONS OF SOME DEGENERATE PARABOLIC EQUATIONS

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Dedicated to the memory of Peter Hess

Abstract. We consider positive solutions u with Dirichlet boundary conditions of the equation

$$u_t = u^p(\Delta u + u)$$

with $p > 1$ on a set Ω which has first eigenvalue less than one. The existence of a continuous solution (smooth inside and unique for $p \geq 2$) is proved, provided $0 < c_0 \leq \operatorname{dist}(x, \partial\Omega)^{-1} u(x, 0) \leq c_1$. Furthermore the solution blows up before $T^* = c(n, p, \Omega) \cdot c_0^{-p}$.

Let us consider the following nonlinear degenerate parabolic diffusion equation

$$u_t = u^p(\Delta u + u) \tag{1}$$

on a bounded domain $\Omega \subset \mathbb{R}^n$ (with $\partial\Omega$ of class $C_{2+\alpha}$) under Dirichlet conditions

$$u_{|\partial\Omega} = 0 \tag{2}$$

and

$$u(x, 0) = \Phi(x), \tag{3}$$

where we assume $\Phi(x) \in C_3(\overline{\Omega})$, $\Phi_{|\partial\Omega} = 0$, $\Phi(x) \geq c_0 \operatorname{dist}(x, \partial\Omega)$ with some $c_0 > 0$ and $\Delta\Phi + \Phi \geq 0$.

The case $p = 2$ was considered in a fundamental paper by Friedman - McLeod [2]. They observed a decisive dependence of this problem on the size of the domain.

Denoting by Θ the first (positive, normalized) eigenfunction of $-\Delta$ under Dirichlet conditions ($\Delta\Theta + \lambda_1\Theta = 0$, $\Theta_{|\partial\Omega} = 0$) they established the existence of a *global* solution for *small* domains (meaning $\lambda_1 > 1$), while for *large* domains ($\lambda_1 < 1$) the existence of a local solution was proved, which performanced a blow-up after a finite time.

In 1992 Gage [3] considered the blow-up case again for $p = 2$, gave an estimate for the blow-up time and made a more detailed study of the set, where the solution blows up.

We shall consider in this paper the general case $p > 1$ and prove the following:

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