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TRANSVERSAL AND NONTRANSVERSAL INTERSECTIONS OF STABLE AND UNSTABLE MANIFOLDS IN REACTION DIFFUSION EQUATIONS ON SYMMETRIC DOMAINS

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To the memory of Peter Hess

Abstract. Scalar reaction-diffusion equations on a ball in $\mathbb{R}^N$, $N \geq 2$, with radially symmetric nonlinearities and Dirichlet boundary condition are considered. If the nonlinearity is nonincreasing in the radial variable (in particular if it is independent of it) it is proved that the stable and unstable manifolds of any two nonnegative equilibria intersect trasversally. The crucial property used in the proof is that the unstable manifold of a positive equilibrium consists of radially symmetric functions. Then, an equation is constructed that admits two radially symmetric equilibria whose invariant manifolds intersect nontransversally. In the appendix, examples of spatially homogeneous equations with positive equilibria with high Morse indices are given.

1. Introduction. Consider the following semilinear parabolic problem

$$u_t = \Delta u + f(u) \quad \text{on } \Omega \tag{1.1}$$

$$u|_{\partial\Omega} = 0, \tag{1.2}$$

where Ω is the unit ball $\{x \in \mathbb{R}^N : |x| < 1\}$ in the Euclidian space $\mathbb{R}^N$, $N \geq 2$, and $f : \mathbb{R} \to \mathbb{R}$ is of class C^2.

This problem defines a local semiflow on an appropriate Banach space X. Specifically, we choose X to be the Sobolev space $X = W_0^{1,p}(\Omega)$ with $p > N$. Then X imbeds continuously in $C^\alpha(\bar{\Omega})$ for some $\alpha > 0$, hence (1.1),(1.2) is well posed on X by the theory of [21]. For $u_0 \in X$, we denote by $u(t, \cdot, u_0)$ the maximal solution of (1.1), (1.2) (in the sense of [21], hence a classical solution) satisfying the initial condition $u(0) = u_0$. Let $S(t)$, $t \geq 0$, be the local semiflow of (1.1), (1.2); that is, $S(t)u_0 = u(t, \cdot, u_0)$ when the latter is defined.

Though problems of the form (1.1), (1.2) present one of the basic classes of infinite-dimensional dynamical systems and have been widely studied, their dynamics have not been fully understood. On one hand, it is known that $S(t)$ has the gradient-like structure, the usual energy functional being its Lyapunov function. Hence all bounded trajectories of (1.1), (1.2) approach a set of equilibria. On the other hand, it has not been decided whether, in general, each such trajectory must converge to just

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