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P-ARCS IN STRONGLY MONOTONE DISCRETE-TIME DYNAMICAL SYSTEMS

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Dedicated to the memory of Professor Peter Hess (1941–1992)

Abstract. Strongly monotone discrete-time (semi)dynamical systems are generated by second order parabolic partial differential equations periodic in time, complemented with boundary conditions admitting the strong maximum principle. It has been recently proved by P. Hess and P. Poláčik that, under some additional conditions, the set of periods of periodic points which are not linearly unstable is bounded from above. In the present paper we use the above theorem, together with results by P. Takáč, to state and prove several propositions about classification of points and genericity of some properties. A main tool in the proofs is the concept of nondegenerate p-arc, that is, a simply ordered invariant set diffeomorphic to [0, 1]. The paper is divided into four sections. Sections 1 and 2 contain preliminary results. In Section 3 basic properties of p-arcs are discussed. In Section 4 we investigate generic behavior, that is, long-time behavior of points belonging to some open dense subset of the phase space.

The theory of strongly monotone dynamical systems with continuous time, initiated independently by M.W. Hirsch ([21]) and H. Matano ([29], [30]), has undergone a mighty development since then, whether from the theoretical point of view (see e.g. [44], [45]) or from the point of view of applications (see e.g. [10], or [16]). In the discrete time case however, only quite recently a general abstract theory was developed by P. Takáč in [46] and [47] (but see the earlier papers [20], [1], [3], and for the finite-dimensional case [43]). At the same time, new results have been obtained by P. Hess, P. Poláčik and I. Tereščák ([38] and [19]).

In the present paper we generalize and improve on some results from [47] as well as from [38], assuming that the strongly monotone mapping considered is of class C^1. We introduce the concept of nondegenerate p-arcs (see Section 3), which are defined to be simply ordered invariant sets diffeomorphic to [0, 1]. (In a sense they can be considered "duals" of Takáč's d-hypersurfaces, see [47]). If the equilibria contained in a nondegenerate p-arc satisfy some spectral property then the p-arc is an attractive normally hyperbolic C^1 one-dimensional manifold with boundary, which allows us to use such tools as forward invariant laminations (foliations). In

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