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ON SOLUTION CONTINUA OF SUPERCRITICAL QUASILINEAR ELLIPTIC PROBLEMS

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Dedicated to the memory of Peter Hess

Abstract. In this paper, we prove uniqueness of positive solutions (for certain values of the parameter λ) to quasilinear problems of the form $\operatorname{div}\{g(x, \nabla u)\} + \lambda f(x, u) = 0$ with zero Dirichlet data on a bounded starlike domain in $\mathbb{R}^n$, $n \geq 3$. Here g is assumed to be a vector function whose norm grows subcritically in $|\nabla u|$. The function f grows supercritically in u.

1. Introduction. This paper is concerned with quasilinear elliptic problems of the form

$$\operatorname{div}\{g(x, \nabla u)\} + \lambda f(x, u) = 0, \quad x \in \Omega, \quad u = 0, \quad x \in \partial\Omega, \tag{1}$$

where the domain Ω is a bounded starlike set in $\mathbb{R}^n$, $n \geq 3$ with a smooth boundary: $\partial\Omega \in C^{2,\alpha}$. Problem (1) may arise as the steady state formulation of a related parabolic problem [2] or may arise directly from problems in differential geometry [3] and physics [5]. An important subclass of problems is obtained when $g_j(x, \nabla u) = \sum_i a_{ij}(x)\partial u/\partial x_i$; in this case, (1) is called a *semilinear* problem.

The function $f(x, u)$ is assumed to be nonnegative and the partial derivative $f_u(x, u)$ is assumed to be strictly positive. A second growth assumption is placed on f, namely that f grows supercritically (defined below). Concerning g, it will be assumed that it grows subcritically (also defined below). Several good review articles are concerned with problems of this type [1, 17, 13].

A model problem for (1) is the Gelfand problem:

$$\Delta u + \lambda e^u = 0, \quad u \in \Omega, \quad u = 0, \quad u \in \partial\Omega. \tag{2}$$

This problem arises in combustion theory as the steady-state equation for a semilinear reaction-diffusion problem [2]. The main goal of this paper is to demonstrate that the essential solution continuum behavior found for the Gelfand problem when Ω is a ball can be found in solution continua of (1) where Ω is a starlike domain.

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