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ON AN ELLIPTIC BOUNDARY VALUE PROBLEM IN A HALF SPACE

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Dedicated to the late Peter Hess

Abstract. In the spectral theory for non-selfadjoint elliptic bound value problems involving an indefinite weight function, there arises for the case of second order operators a boundary value problem in a half-space which is of fundamental importance to the theory. In this work we fix our attention upon such a half-space problem, and by means of Fourier transforms and Michlin's multiplier theorem, we prove the existence, uniqueness, and an a priori bound for the solution.

1. Introduction. We are all aware of the significant contribution made by Peter Hess to the spectral theory for elliptic boundary value problems involving an indefinite weight function (cf. [13–16]). Of particular interest to Peter were the non-selfadjoint problems, with a typical example being given by the oblique derivative problem for a second order operator. Stimulated by our many conversations concerning such problems, the author was led in earlier works to initiate an investigation into the associated spectral theory, and accordingly it is the object of this work to continue with that investigation.

In the spectral theory for non-selfadjoint elliptic boundary value problems involving an indefinite weight function, there arises for the case of second order operators a boundary value problem that is of fundamental importance to the theory and which is of the form

$$L(x_n, D, q)u = \sum_{|\alpha|=2} a_\alpha D^\alpha u + q^2 \chi(x_n) u = f(x) \ \text{ in } \mathbb{R}^n_+, \tag{1.1}$$

$$B(D)u = h(x') \ \text{ on } x_n = 0, \tag{1.2}$$

where $D = (D_1, \dots, D_n)$, $D^\alpha = D_1^{\alpha_1} \cdots D_n^{\alpha_n}$, $D_j = -i\partial/\partial x_j$, $\alpha = (\alpha_1, \dots, \alpha_n)$ is a multi-index with $|\alpha| = \sum_{j=1}^n \alpha_j$, $x = (x_1, \dots, x_n) = (x', x_n) \in \mathbb{R}^n$, $n \geq 2$, $\mathbb{R}^n_+ = \{(x', x_n) \in \mathbb{R}^n : x_n > 0\}$, $\sum_{|\alpha|=2} a_\alpha D^\alpha$ is an elliptic operator with the a_α being real-valued constants, q is a complex parameter varying in the closed sector $\Sigma : \theta_1 \leq \arg\, q \leq \theta_2 (0 \leq \theta_2 - \theta_1 < \pi)$, $\chi(x_n)$ is a real-valued smooth function

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