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EXPONENTIAL STABILITY, CHANGE OF STABILITY AND EIGENVALUE PROBLEMS FOR LINEAR TIME-PERIODIC PARABOLIC EQUATIONS ON $\mathbb{R}^N$

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In memory of Peter Hess

1. Introduction. The main purpose of this article is to give conditions on a non-negative weight function m which are necessary and sufficient for the zero solution of the linear time-periodic parabolic equation

$$\partial_t u - \Delta u = -m(x,t)u \qquad \text{in } \mathbb{R}^N \times (0,\infty) \tag{1.1}$$

to be exponentially stable and to apply these results to the study of change of stability in parameter dependent time-periodic parabolic problems. Here, Δ denotes the Laplacian in $\mathbb{R}^N$ and the weight-function lies in $C_T^\nu(\mathbb{R}, BUC(\mathbb{R}^N))$, the space of ν-Hölder continuous and T-periodic functions taking values in the space of bounded uniformly continuous functions on $\mathbb{R}^N$ ($\nu \in (0,1)$ and $T > 0$ fixed). Stability is to be understood either with respect to the L_∞- or the L_1-norm and initial conditions are taken in the spaces $C_0(\mathbb{R}^N)$ and $BUC(\mathbb{R}^N)$ or $L_1(\mathbb{R}^N)$, respectively. As it turns out exponential stability will be a property which does not depend on the underlying space.

Our first characterization of exponential stability will be in terms of a quality that m may have or not as an inhomogeneity in the initial value problem

$$\begin{cases} \partial_t u - \Delta u = m(x,t) & \text{in } \mathbb{R}^N \times (0,\infty) \\ u(x,0) = 0 & \text{in } \mathbb{R}^N. \end{cases} \tag{1.2}$$

We can actually prove the following result, generalizing the equivalence of statements (1) and (6) of Proposition 4.19 in the paper of C.J.K. Batty [3] which treats the time-independent case, i.e., the case of Schrödinger semigroups.

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