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PARAMETER DEPENDENCE IN THE b–ϵ MODEL

M. BERTSCH AND R. DAL PASSO
Dipartimento di Matematica, Università di Roma Tor Vergata
Via della Ricerca Scientifica, 00133 Roma, Italy

R. KERSNER
Computer and Automation Institute of the Hungarian Academy of Sciences
P.O. Box 63, H–1518 Budapest, Hungary

Dedicated to the memory of Peter Hess

1. Introduction. We consider the following system of partial differential equations:

$$\text{(I)} \qquad \begin{cases} b_t = \left(\frac{b^2}{\epsilon} b_x\right)_x - \epsilon & \text{in } Q = \{(x,t) : x \in \mathbb{R}, t > 0\} \\ \epsilon_t = \left(\frac{b^2}{\epsilon} \epsilon_x\right)_x - \gamma \frac{\epsilon^2}{b} & \text{in } Q \\ b(x,0) = b_0(x), \quad \epsilon(x,0) = \epsilon_0(x) & \text{for } x \in \mathbb{R}, \end{cases}$$

where b_0 and ϵ_0 are given bounded, nonnegative and continuous real functions on $\mathbb{R}$ with equal support, and where γ is a positive parameter. Subscripts denote partial differentiations.

Problem I arises in the context of the evolution of turbulent bursts and is usually referred to as b–ϵ (or also k–ϵ) model. This semi-empirical model is based on the ideas of Kolmogorov [18] and Prandtl [20]; for an introduction to the physical model we refer to [3], which contains several references to the literature. Here we only mention that $b(x,t)$ represents the turbulent energy density and $\epsilon(x,t)$ is the dissipation rate of turbulent energy.

The main mathematical obstacle to treat Problem I is the appearance of the singular functions b^2/ϵ and ϵ^2/b. In an earlier paper [3] we have shown that if

$$\gamma \geq 1 \tag{1.1}$$

and if there there exist positive constants C_0 and C_1 such that

$$0 \leq C_0\epsilon_0(x) \leq b_0(x) \leq C_1\epsilon_0(x) \quad \text{for } x \in \mathbb{R}, \tag{1.2}$$

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