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FUJITA TYPE PHENOMENA FOR REACTION-DIFFUSION EQUATIONS WITH CONVECTION LIKE TERMS

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Dedicated to the memory of our friend and colleague, Peter Hess

Abstract. We study the long time behavior of nonnegative solutions of the initial-boundary value problem, for $0 < T \leq +\infty$,

$$u_t - \Delta u - \mathbf{b} \cdot \nabla u = u^p \quad \text{in } \mathbb{R}^N \times (0, T), \quad u(\mathbf{x}, 0) = u_0(\mathbf{x}) \geq 0.$$

Here $p > 1$ and $\mathbf{b} = (b_1, \ldots, b_N)$. We extend Fujita's result for the case $\mathbf{b} = \mathbf{0}$. He and others proved that if $1 < p \leq 1 + 2/N$, $T = +\infty$ implies that $u \equiv 0$ while if $p > 1 + 2/N$, for some choices of $u_0 \not\equiv 0$, $T = +\infty$. We consider separately the cases for which $\mathbf{b} = \mathbf{b}(u)$ and $\mathbf{b} = \mathbf{b}(\mathbf{x})$. We also discuss this problem to a limited degree when $\mathbb{R}^N$ is replaced by a cone.

1. Introduction. This note is concerned with the question of the global existence of nonnegative solutions of problems of the type

$$\begin{aligned} u_t - \Delta u - (\mathbf{b}, \nabla u) &= u^p \quad \text{in} \quad D \times (0, T) \\ u &= 0 \quad \text{on} \quad \partial D \times (0, T) \\ u(\mathbf{x}, 0) &= u_0(\mathbf{x}) \geq 0, \quad u(\mathbf{x}, t) \geq 0, \end{aligned} \tag{1.1}$$

where $\mathbf{b} = (b_1, b_2, \ldots, b_N)$, $b_i = b_i(\mathbf{x}, u)$, $p > 1$ and D is either $\mathbb{R}^N$ (and then the boundary condition is dropped) or some other (unbounded)region. Here T is the maximal time of existence, $T \leq +\infty$. When $T = +\infty$ we say u is global. Otherwise we say "u blows up in finite time" and use this as a euphemism for the statement that u is not global in time.

Equation (1.1) can be interpreted as a model for a reaction-diffusion process where u represents the temperature, u^p is a nonlinear source term and the convection $(\mathbf{b}, \nabla u)$ can be caused by an external flow field.

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