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# GAUSSIAN ESTIMATES AND INTERPOLATION OF THE SPECTRUM IN $L^p$

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In memoriam Peter Hess

**Abstract.** It is shown that the spectrum of a uniformly elliptic operator on $L^p(\Omega)$ with Dirichlet or Neumann boundary conditions is independent of $p \in [1, \infty)$.

**0. Introduction.** Let $\Omega \subset \mathbb{R}^N$ be an open set and let $T_p = (T_p(t))_{t\geq 0}$ be consistent $C_0$-semigroups on $L^p(\Omega)$ with generators $A_p$ $(1 \leq p < \infty)$. It is natural to ask whether the spectrum $\sigma(A_p)$ of $A_p$ is independent of $p \in [1, \infty)$. This is not the case in general (see Hempel-Voigt [13], [14], Davies [9, 4.3], Jörgens [15]); here we give a particularly simple example: if

$$(A_p f)(x) = -x\, f'(x)$$

on $L^p(0, \infty)$, then $\sigma(A_p) \cap \sigma(A_q) = \emptyset$ for $p \neq q$; see Section 3.

Our main result is the following: assume that $A_2$ is self-adjoint and $T_2$ satisfies an upper Gaussian estimate. Then $\sigma(A_p)$ is independent of $p \in [1, \infty)$.

Gaussian estimates have been studied extensively; see the books of Davies [9], Robinson [21] and Varopoulos, Saloff-Coste, Coulhon [27]. In particular, if $A_p$ is a self-adjoint second order differential operator with Dirichlet or Neumann boundary conditions such estimates are known to hold, and thus, by our result, $\sigma(A_p)$ is independent of $p \in [1, \infty)$. Also, if $A_p = \Delta - V$ is a Schrödinger operator on $L^p(\mathbb{R}^N)$ Gaussian estimates have been established (see [24]). Thus our result generalizes that of Hempel-Voigt [14] who prove $p$-independence in that case. In fact, we use the same strategy as Hempel and Voigt, and show that the resolvent consists of integral operators whose kernels can be estimated. But instead of regularizing by considering powers of the resolvent as in [14] we regularize by the semigroup (see (6.9)). This simplifies the proof and gives more precise results : we obtain that the resolvent consists entirely of regular integral operators (cf. [4]).

The paper is organized as follows. In Section 1 we consider the much easier case where $\Omega$ is bounded. Consistency of the resolvents is studied in Section 2. In fact,

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