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ON A CLASS OF RESONANT PROBLEMS AT HIGHER EIGENVALUES*

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Introduction. In this note we present existence and multiplicity results for resonant problems at higher eigenvalues of $-\Delta$ on $H_0^1(\Omega)$. More precisely, we consider the problem

$$-\Delta u = \lambda_k u + g(x,u) \quad \text{in } \Omega\ , \quad u = 0 \text{ on } \partial\Omega, \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 1$) is a bounded domain, $\lambda_k > \lambda_{k-1}$ ($k \geq 2$) is an eigenvalue of the problem $-\Delta u = \lambda u$ in Ω, $u = 0$ on $\partial\Omega$, and $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function with $\sup_{|s|\leq r} \mid g(x,s) \mid = g_r(x) \in L^q(\Omega)\ \forall r > 0$, for some $q \geq 2N/(N+2)$ if $N \geq 3$ (any $1 \leq q \leq \infty$ if $N = 1, 2$). We will assume that g satisfies the basic assumption

$$\frac{g(x,s)}{s} \longrightarrow 0 \ \text{ as } \ \mid s \mid \to \infty, \text{ uniformly for a.e. } x \in \Omega, \tag{g_1}$$

characterizing (P) as a resonant problem at λ_k.

One of the main points in this article is to explore the fact that higher eigenvalues always change sign. Our first result uses this fact to show that the corresponding functional

$$I(u) = \int_\Omega \frac{1}{2}(\mid \nabla u \mid^2 - \lambda_k u^2)dx - \int_\Omega G(x,u)\,dx, \quad u \in H_0^1(\Omega), \tag{1}$$

satisfies the Palais-Smale condition under an one-sided *nonquadraticity* assumption at $-\infty$ for the potential $G(x,s) = \int_0^s g(x,t)dt$ (cf. [6]). The only further restriction we shall need in this case is to assume a stronger version of (g_1) for $s > 0$, namely,

$$g(x,s) \to 0 \quad \text{as } s \to \infty, \text{ uniformly for a.e. } x \in \Omega. \tag{g_2}$$

It should be noted that (g_2) allows a rather general (sublinear) behavior of $G(x,s)$ for $s > 0$, including *strongly resonant* situations ($G(x,s)$ bounded) as well as an unbounded oscillatory behavior for $G(x,s)$.

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