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NEW NONEXISTENCE RESULTS FOR ELLIPTIC EQUATIONS WITH SUPERCRITICAL NONLINEARITY

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1. Introduction. In this paper we consider the problem

$$(*)\qquad \begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega \\ u = 0 \text{ on } \partial\Omega;\ u \not\equiv 0 & \text{in } \Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^n$ with $n \geq 3$ and f is a continuous function such that $f(0) = 0$ and

$$f(t) \sim t|t|^{p-2} \text{ for } t \to \pm\infty, \text{ with } p \geq \frac{2n}{n-2}$$

($\frac{2n}{n-2}$ is the critical exponent for the Sobolev embedding $H^{1,2}(\Omega) \subset L^p(\Omega)$). It is well known (see Pohozaev [27]) that Problem $(*)$ has no solution if Ω is starshaped and, for instance, $f(t) = t|t|^{p-2}$ with $p \geq \frac{2n}{n-2}$. On the other hand, a remarkable result of Bahri and Coron (see [1]) guarantees that, if Ω has nontrivial topology (in a suitable sense), then Problem $(*)$ with $f(t) = t|t|^{p-2}$ and $p = \frac{2n}{n-2}$ has at least one positive solution.

Other existence, nonexistence and multiplicity results of positive solutions for Problem $(*)$ with $f(t) = t|t|^{p-2}$ and $p = \frac{2n}{n-2}$, under suitable assumptions on the shape of Ω, have been obtained by several authors (see [9], [11], [10], [8], [20], [21], [24], [30]). Hence, it is natural to ask whether the result of Bahri-Coron [1] can be extended to the case $p > \frac{2n}{n-2}$ (for other problems concerning the case $p > \frac{2n}{n-2}$ see also [6], [28], [29], [16], [17], [18], [19]).

A result proved in [26] shows that the nontriviality of the topology of Ω in the sense of Bahri-Coron [1] does not guarantee the existence of solutions for Problem $(*)$ with $f(t) = t|t|^{p-2}$ for every $p > \frac{2n}{n-2}$ (this question has been posed by Rabinowitz, as Brezis reports in [4]). More precisely, in [26] it is proved that, if $p > \frac{2(n-k)}{n-k-2}$ (with $1 \leq k \leq n-3$), then there exists a bounded domain $\Omega \subset \mathbb{R}^n$, homotopically equivalent to the k-dimensional sphere S_k (hence topologically nontrivial in the sense of [1]), such that $(*)$ has no solution when $f(t) = t|t|^{p-2}$.

In this paper we obtain more general and complete results, which enable us to state, in particular, the following proposition (see also Theorem 3.1 and Applications 3.2 for more precise and general statements).

Proposition 1.1. *For every positive integer k such that $1 \leq k \leq n-3$ there exists a bounded domain Ω homotopically equivalent to the k-dimensional sphere S_k (hence topologically nontrivial) such that Problem $(*)$ with $f(t) = t|t|^{p-2}$ has no solutions*

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