Differential and Integral Equations, Volume 8, Number 1, January 1995, p. 224.

Erratum

"On the asymptotic behavior of minimizers of the Ginzburg-Landau model in 2-dimensions," by "Michael Struwe," Differential and Integral Equations, Volume 7, Number 6 (1994), 1613–1624.

As stated, Proposition 3.4 may not be applied in the proof of Proposition 3.3 because the assumption

$$\frac{1}{\epsilon^2}\int_{\Omega\cap B_R(0)}\left(1-|u_\epsilon|^2\right)^2 dx \le K \tag{1}$$

made in Proposition 3.4 has only been verified for $R \le 5\epsilon^{1/4}$; see Lemma 3.1. The correct statement of Proposition 3.4 is the following:

Proposition 3.4′. *Let $\epsilon < R_0 < R \le R_1$ and suppose $\hat{u} \in H^1_g$ satisfies $|\hat{u}| \le 1$ in Ω, $|\hat{u}(x)| \ge \frac{1}{2}$ in A_{R,R_0} and the estimates*

$$\frac{1}{\epsilon^2}\int_{\Omega\cap B_{\epsilon^{1/4}}(0)}\left(1-|\hat{u}|^2\right)^2 dx \le K, \tag{2}$$

as well as

$$E_\epsilon(\hat{u}) \le K|\ln\epsilon| + K. \tag{3}$$

Then there holds

$$\int_{A_{R,R_0}} |\nabla\hat{u}|^2 dx \ge 2\pi\hat{d}^2\ln\left(\tfrac{R}{R_0}\right) - C\hat{d}^2,$$

where $C = C(\Omega, g, K)$ and where $\hat{d}$ is the topological degree of $\hat{u}$, restricted to $\partial(\Omega\cap B_R(0)) \cong S^1$.

The proof of Proposition 3.4′ is almost identical to the proof of Proposition earlier. We only need to modify the estimates for the error terms I_2 and

$$I_4 = \int_{\bar{A}_{R,R_0}} (1-\rho^2)\frac{\hat{d}^2}{r^2}\,dx.$$

We may assume $R \ge \epsilon^{1/4}$. Then we split

$$I_4 = \int_{\bar{A}_{R,\epsilon^{1/4}}} \cdots + \int_{\bar{A}_{\epsilon^{1/4},R_0}} \cdots = I_5 + I_6$$

and estimate $I_6 \le \hat{d}^2(\pi K)^{1/2}$ as before, while by Cauchy-Schwarz

$$I_5 \leqq \frac{1}{\epsilon^{1/2}}\int_\Omega (1-\rho^2)\hat{d}^2 dx \le \hat{d}^2(\mu(\Omega)\cdot\epsilon E_\epsilon(\hat{u}))^{1/2} \le C\hat{d}^2.$$

Similarly,

$$\begin{aligned}|I_2| = 2\Big|\int_{\bar{A}_{R,R_0}} (1-\rho^2)\frac{\hat{d}}{r^2}\frac{\partial\psi}{\partial\vartheta}\,dx\Big| &\le 4\int_{\bar{A}_{R,R_0}} (1-\rho^2)^2\frac{\hat{d}^2}{r^2}\,dx + \frac{1}{4}\int_{\bar{A}_{R,R_0}} |\nabla\psi|^2 dx\\ &\le 4I_4 + \frac{1}{4}\int_{\bar{A}_{R,R_0}} |\nabla\psi|^2 dx \le C\hat{d}^2 + \frac{1}{4}\int_{\bar{A}_{R,R_0}} |\nabla\psi|^2 dx,\end{aligned}$$

and the proof may be completed as before.

By a different method, it is possible to prove that, in fact, assumption (1) is satisfied for all R and that Proposition 3.4 can be applied in its original form. The simple argument given above was suggested to me by M.-C. Hong.

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