Differential and Integral Equations, Volume 8, Number 1, January 1995, pp. 105 – 130.

THE CAUCHY PROBLEM FOR A GENERALIZED ZAKHAROV SYSTEM

Corinne Laurey
Laboratoire d'Analyse Numérique, CNRS et Université Paris-Sud, Bât 425, 91405, Orsay, France

(Submitted by: J.L. Bona)

1. Introduction. We consider the generalized Zakharov system for $t \in \mathbb{R}^+$ and $x \in \mathbb{R}^d$ $(d = 2, 3)$,

$$\begin{cases} iE_t + \operatorname{grad}\operatorname{div} E - \alpha \operatorname{rot}\operatorname{rot} E - nE + i(E \wedge B) = 0 \\ n_{tt} - \Delta(n + |E|^2) = 0 \\ \Delta B - i\eta \operatorname{rot}\operatorname{rot}(E \wedge \bar{E}) + A = 0, \end{cases} \tag{1.1}$$

with the initial data

$$E(0,x) = E_0(x), \quad n(0,x) = n_0(x), \quad n_t(0,x) = n_1(x), \tag{1.2}$$

where $E(t,x)$ is a vector valued function from $\mathbb{R}^+ \times \mathbb{R}^d$ into $\mathbb{C}^d$, $n(t,x)$ is a function from $\mathbb{R}^+ \times \mathbb{R}^d$ to $\mathbb{R}$, $B(t,x)$ is a vector-valued function from $\mathbb{R}^+ \times \mathbb{R}^d$ into $\mathbb{R}^d$ and A has either the form

(A_1) $A = \beta B$, β a non positive constant, or

(A_2) $A = -\gamma \frac{\partial}{\partial t} \int_{\mathbb{R}^d} \frac{B(t,y)}{|x-y|^2}\, dy$,

and in the second case, the initial data $B(0,x) = B_0(x)$ is added to (1.2).

This system describes the spontaneous generation of a magnetic field in a cold plasma (case A_1) or in a hot plasma (case A_2) [8]. E denotes the slowly varying complex amplitude of the high-frequency electric field, B the self-generated magnetic field, n the fluctuation of the electron density from its equilibrium, and α is a nonnegative constant, generally $\alpha \geq 1$. Omitting the magnetic field B, we recover the system describing Langmuir's turbulence [5], [16],

$$\begin{cases} iE_t + \operatorname{grad}\operatorname{div} E - \alpha \operatorname{rot}\operatorname{rot} E + nE = 0 \\ n_{tt} - \Delta(n + |E|^2) = 0. \end{cases} \tag{1.3}$$

Concerning (1.3), C. and PL. Sulem proved in [14] the global existence of a weak solution, for small initial data in two and three dimensions, supposing in particular $n_1 \in \dot{H}^{-1}(\mathbb{R}^d)$ ($\dot{H}^{-1}(\mathbb{R}^d)$ denotes the homogeneous Sobolev space, $\dot{H}^{-1}(\mathbb{R}^d) = \{u \in S'(\mathbb{R}^d) : \frac{1}{|\xi|}\widehat{u}(\xi) \in L^2(\mathbb{R}^d)\}$). With the same assumptions, they also established local existence and uniqueness of a smooth solution $(E,n) \in L^\infty(0,T; H^m(\mathbb{R}^d)) \times L^\infty(0,T; H^{m-1}(\mathbb{R}^d))$ for $m \geq 3$. Moreover, the solution was shown to be globally defined in one spatial dimension [14], and in two spatial dimensions for small initial data (H. and S. Added [1]).

Similar results, using different methods, have been obtained by S.H. Schochet and M.I. Weinstein [12], for the nonlinear Schrödinger limit of (1.3). T. Ozawa and Y.

Received March 1992, in revised form March 1993.
AMS Subject Classifications: 35Q55, 35A07.