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THE PRINCIPAL EIGENCURVE FOR THE p-LAPLACIAN

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1. Introduction. Over the years, equations of the form

$$Au - \lambda Bu = \mu Cu \tag{1.1}$$

have proved very useful, especially in the case where A and B are linear operators and C is the identity. One may cite early work by Richardson and others on eigencurves for Sturm–Liouville equations (1.1), i.e., sets of (λ, μ) satisfying (1.1) with $u \neq 0$ [24]. This advanced considerably our understanding of the "indefinite weight" problem, a topic which has blossomed recently, cf. the references in Kaper and Zettl [15], Mingarelli [21], and Fleckinger and Lapidus [12]. Not only is (1.1) useful in its own right for, say separation of variables [20] and bifurcation [7] problems, but it provides very useful insight into the structure of the operator pencil $A - \lambda B$, as in the general linear perturbation theory of say Kato [17, e.g., p. 388]. While generalizations of Sturm–Liouville equations have appeared, much of the work has been devoted to second order self-adjoint linear operators, often via abstract linear operator and quadratic form theory.

It is our purpose here to initiate a similar investigation for certain nonlinear operators. Specifically, A will be the p-Laplacian, so $A = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, on a bounded domain $\Omega \subset \mathbb{R}^N$, and we shall take $Cu = |u|^{p-2}u$. We remark that the investigation of the p-Laplacian originates from a variety of practical fields, see e.g., Diaz [11] and Lions [19] for more physical background, and has received growing attention lately, see Atkinson [3], Azorezo and Alonso [4], Otani and Teshima [22] and Szulkin [26] for instance.

The comparative paucity of techniques for nonlinear operators forces us to use the differential equations, and not just abstract homogeneous monotone operators, although we do make some use of variational principles involving pth power (instead of quadratic) forms. In this paper we shall be concerned with the "principal" eigenvalue $\mu_1 = \mu_1(\lambda)$, i.e., for which a positive solution $u = u(\lambda)$ exists on Ω.

In Section 2 we establish basic properties like continuity and differentiability of μ_1 and u, employing standard variational techniques to obtain existence of the eigenvalue μ_1 and eigenfunction u. Section 3 concerns asymptotics of μ_1, μ_1' and u, and in Section 4 we admit perturbation by a (not necessarily homogeneous) nonlinear $f(x, u)$. We focus on the continuous dependence of μ_1 on λ. We note

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