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A DIRECT APPROACH TO INFINITE DIMENSIONAL HAMILTON–JACOBI EQUATIONS AND APPLICATIONS TO CONVEX CONTROL WITH STATE CONSTRAINTS*

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1. Introduction. For several reasons convex optimal control plays a special role in the theory of Distributed Parameter Systems. For instance, convex control is one of the few examples of nonlinear control problems possessing a smooth value function. As well known, this fact is essential to constructing optimal feedback strategies by the Dynamic Programming approach. Moreover, such a function can obtained by a direct method, solving a first order nonlinear partial differential equation, the so–called Hamilton–Jacobi–Bellman equation (see [1]).

The present paper is devoted to the analysis of Hamilton–Jacobi equations, when related to control problems with constraints on the state.

To fix ideas, let X and U be separable real Hilbert spaces, $0 \le t \le T$, and consider the problem of minimizing, overall controls $u \in L^2(t,T;U)$, the cost functional

$$f(y(T)) + \int_t^T \big[\frac{1}{2}\|u(s)\|_U^2 + g(y(s))\big]ds, \tag{1.1}$$

where $y \in C([t,T];X)$ is the mild solution of the state equation

$$\begin{cases} y'(s) = Ay(s) + Bu(s), \quad t \le s \le T \\ y(t) = x. \end{cases} \tag{1.2}$$

Here, we assume the following conditions, that are typical in convex control:

$$\begin{array}{ll} \text{(i)} & f, g : X \to [0, +\infty \text{ are continuous and convex functions such that} \\ & f(0) = 0 = g(0); \\ \text{(ii)} & A : D(A) \subset X \to X \text{ is the infinitesimal generator of a } C_0\text{-semigroup} \\ & e^{tA} \text{ satisfying } \|e^{tA}x\|_X \le e^{\alpha t}\|x\|_X \text{ for some } \alpha \in \mathbb{R}; \\ \text{(iii)} & B \in \mathcal{L}(U, X). \end{array} \tag{1.3}$$

A control $\overline{u}$ at which the above functional attains its minimum is said to be *optimal*. The pair $\{\overline{y}, \overline{u}\}$, where $\overline{y}$ is the corresponding solution of equation (1.2), is called an optimal pair.

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