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ON A GENERAL CLASS OF BIRKHOFF-REGULAR EIGENVALUE PROBLEMS

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1. Introduction. In this article we consider boundary eigenvalue problems of the form

$$\ell(y) = y^{(n)} + \sum_{\nu=1}^{n} p_\nu(x) y^{(n-\nu)} = \lambda y,\ x \in [0,1], \tag{1.1}$$

$$U_\nu(y) = U_{\nu 0}(y) + U_{\nu 1}(y) = 0,\ 1 \le \nu \le n. \tag{1.2}$$

We assume that $p_\nu \in L[0,1],\ 1 \le \nu \le n$, and that the boundary conditions (1.2) are normalized; this means that

$$U_{\nu 0}(y) = \alpha_\nu y^{(k_\nu)}(0) + \sum_{j=0}^{k_\nu - 1} \alpha_{\nu j} y^{(j)}(0), \quad U_{\nu 1}(y) = \beta_\nu y^{(k_\nu)}(1) + \sum_{j=0}^{k_\nu - 1} \beta_{\nu j} y^{(j)}(1), \tag{1.3}$$

where $\alpha_{\nu j}, \beta_{\nu j} \in \mathbb{C}$,

$$|\alpha_\nu| + |\beta_\nu| > 0 \ \text{ for } \ 1 \le \nu \le n,$$

$n-1 \ge k_1 \ge k_2 \ge \cdots \ge k_n \ge 0$ with $k_{\nu+2} < k_\nu$ for $1 \le \nu \le n-2$, and where $k_0 := \sum_{\nu=1}^{n} k_\nu$ is minimal with respect to all equivalent boundary conditions. k_ν is called the *order* of U_ν.

The study of nonself-adjoint eigenvalue problems generated by n^{th}-order differential expressions (1.1) with smooth coefficients p_ν and by two-point boundary conditions (1.2) was originated by Birkhoff. In [2] Birkhoff proved asymptotic estimates for a fundamental system of solutions of (1.1), then, in [3], he introduced the class of (Birkhoff-)regular boundary conditions and he obtained sufficient conditions for the pointwise convergence of the expansion of a function f into a series of eigen- and associated functions (e.a.f.'s) of (1.1), (1.2). The corresponding series are called Birkhoff-series.

Some years later Tamarkin ([16]) and Stone ([15]) found under more general hypotheses that the expansion of a function $f \in L[0,1]$ into a series of e.a.f.'s of regular

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