Differential and Integral Equations, Volume 8, Number 6, July 1995, pp. 1533 – 1554.

THE EFFECT OF THE DOMAIN GEOMETRY ON THE NUMBER OF POSITIVE SOLUTIONS OF NEUMANN PROBLEMS WITH CRITICAL EXPONENTS

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(Submitted by: Klaus Schmitt)

1. Introduction. Let $\Omega \subset R^N (N \geq 3)$ be a bounded domain with smooth boundary. We are concerned with the existence of positive solutions of the problem

$$(\mathrm{I})_\lambda \qquad \begin{cases} -\Delta u + \lambda u = |u|^{p-1}u & \text{in } \Omega \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\lambda > 0$ is a constant, ν is the unit outer normal to $\partial\Omega$, and $p = \frac{N+2}{N-2}$ is the critical Sobolev exponent.

There have been several papers [2, 5, 6, 19, 20, 24] recently concerning the existence of positive solutions for problems like $(\mathrm{I})_\lambda$, trying to assess the influence of the domain on the existence of positive solutions. Existence results for nonconstant positive solutions of $(\mathrm{I})_\lambda$ for λ large have been established(see e.g. [2] and [24]). In this paper, we are not only concerned with the existence of solutions but also want to study the effect of the domain geometry (actually the geometry of the boundary of the domain) on the structure of the set of solutions for $(\mathrm{I})_\lambda$, especially on the number of solutions, and we shall show how the number of solutions of $(\mathrm{I})_\lambda$ is affected by the topology and geometry of $\partial\Omega$. We shall, by imposing some geometrical condition on $\partial\Omega$, establish a multiplicity result which gives a lower bound on the number of nonconstant positive solutions in terms of the topology of $\partial\Omega$. Solutions related to the local properties of $\partial\Omega$ have been obtained recently in [4] and [29]. We want to point out that our results in this paper supplement the work in [4] and [29] in the sense that only global properties of $\partial\Omega$ are involved here.

For the subcritical case (i.e., $1 < p < \frac{N+2}{N-2}$) we have (cf. [25] and [26]) obtained similar results without assuming any geometrical conditions on $\partial\Omega$.

Let $H(x)$ be the mean curvature of $\partial\Omega$ at $x \in \partial\Omega$ with respect to the unit outward normal to $\partial\Omega$. Assume

$$\partial\Omega = \Gamma_1 \cup \Gamma_2 \cup \cdots \cup \Gamma_k, \tag{1.1}$$

where each Γ_i is an $(N-1)$-dimensional connected closed manifold and disjoint with $\Gamma_j,\ j \neq i$. Define, for $\delta > 0$,

$$\Gamma_i^\delta = \{x \in \Gamma_i : H(x) \geq \delta\}, \quad i = 1, 2, \ldots, k, \tag{1.2}$$

Received for publication October 1993.
*Supported in part by an NSF grant (DMS-9201283) and a faculty research grant at Utah State University (SM-11122).

AMS Subject Classification: 35B30, 35J20, 35J25, 58F14.