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EXISTENCE AND NONEXISTENCE OF POSITIVE SINGULAR SOLUTIONS FOR SEMILINEAR ELLIPTIC PROBLEMS WITH APPLICATIONS IN ASTROPHYSICS

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1. Introduction. Stationary radially symmetric models in stellar dynamics have been studied extensively. Eddington [11] in 1915 introduced the equation

$$\Delta u + \frac{e^{2u}}{1+|x|^2} = 0 \text{ in } \mathbb{R}^3$$

to study the gravitational potential u of a globular cluster of stars. Fifteen years later, Matukuma [24, 25] proposed the equation

$$\Delta u + \frac{u^p}{1+|x|^2} = 0 \quad \text{in } \mathbb{R}^3$$

to improve Eddington's model. Here $u > 0$ represents the gravitational potential, $\rho = (4\pi)^{-1}(1+|x|^2)^{-1}u^p$ is the density and $\int_{\mathbb{R}^3} \rho dx$ represents the total mass. (See also Ni and Yotsutani [34]). In 1972, Peebles [35, 36] gives for the first time a derivation of the steady-state distribution of stars near a massive collapsed object, such as a black hole, located at the center of a globular cluster. The same year, Peebles [35] motivated the observer and theoretician with the title of his paper, "Black holes are where you find them" and concluded, that "there can be no conclusions until we find a black hole". Since then, a great deal has been written about black holes by astrophysicists (see the recent review article by Shapiro [42]). However, the question of the existence of a black hole in a globular cluster is still open. Hubble Space Telescope (HST) observations of globular cluster cores should improve the observational basis for confirming or denying the presence of massive black holes in globular clusters (see, e.g., Cohen [10]). Core collapse does occur; for instance, using the HST, Bendinelli et.al. [5, May 1993] have

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