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SLOW DECAY AND THE HARNACK INEQUALITY FOR POSITIVE SOLUTIONS OF $\Delta u + u^p = 0$ in $\mathbf{R}^n$

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1. Introduction. In a recent paper [9], the author investigated the question of symmetry of positive solutions of the Lane-Emden equation

$$\Delta u + u^p = 0, \qquad \text{in } \mathbf{R}^n,\ p > 1,\ n > 2. \tag{I}$$

Let

$$l = \frac{n+2}{n-2}, \quad m = \begin{cases} \infty, & n = 3, \\ \frac{n+1}{n-3}, & n > 3, \end{cases}$$

the Sobolev exponent for dimensions n and $n-1$ respectively. The following result was proved in [9], using the Alexandroff-Serrin moving-plane method and an asymptotic expansion at infinity.

Theorem 1.1. *Let u be a positive solution of* (I). *Suppose that there exists a constant $M = M(u) > 0$ such that*

$$|x|^\alpha u(x) \le M, \tag{1.1}$$

where

$$\alpha = \frac{2}{p-1}, \qquad \lambda = \Big(\alpha(n-2-\alpha)\Big)^{\alpha/2}.$$

Then u is necessarily radially symmetric about some point $x_0 \in \mathbf{R}^n$, provided that

$$l < p < m. \tag{1.2}$$

Remark. Equation (I) admits infinitely many solutions satisfying (1.1); see [1].

In this paper, our main purpose is to weaken the *slow decay* assumption (1.1). In fact, a large class of solutions satisfies (1.1). Consider the function class

$$Z = \Big\{ u > 0,\ u \in C^1(\mathbf{R}^n) : \nabla u(x) \cdot x = r u'(x) \le \Lambda u,\ |x| \ge \Lambda \Big\}$$

for some $\Lambda = \Lambda(u) > 0$. For solutions of (I) in Z, a local Harnack inequality at infinity is obtained, which implies the needed slow decay estimates. We have the following result.

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