Differential and Integral Equations, Volume 8, Number 6, July 1995, pp. 1317 – 1329.

ON A SEMILINEAR ELLIPTIC SYSTEM

Ph. Clément
Department of Mathematics and Informatics, TU Delft, The Netherlands

R.C.A.M. van der Vorst
Mathematical Institute, Leiden University, The Netherlands

(Submitted by: L.A. Peletier)

1. Introduction. In [9, 7, 11] the following system was studied:

$$\text{(I)} \quad \begin{cases} -\Delta v = H_u(u,v), & \text{in } \Omega, \quad (1.1) \\ -\Delta u = H_v(u,v), & \text{in } \Omega, \quad (1.2) \\ u = v = 0, & \text{on } \partial\Omega, \quad (1.3) \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$ (to be specified later) and $H \in C^1(\mathbb{R}^2;\mathbb{R})$ satisfies appropriate growth conditions. Solutions were obtained by means of a variational principle. Indeed (1.1) and (1.2) are the Euler-Lagrange equations of the Lagrangian

$$\mathcal{L}(z) = \int_\Omega \nabla u \nabla v - \int_\Omega H(u,v). \tag{1.4}$$

Suppose H_u and H_v satisfy the growth conditions

$$|H_u(u,v)| \le c_1 + c_2|u|^p + c_3|v|^{(q+1)\frac{p}{p+1}}, \quad |H_v(u,v)| \le c_4 + c_5|v|^q + c_6|u|^{(p+1)\frac{q}{q+1}}, \tag{1.5}$$

with

$$p,q > 1, \quad \frac{1}{p+1} + \frac{1}{q+1} > \frac{N-2}{N}, \quad N \ge 1, \tag{1.6}$$

with c_1-c_6 positive constants. Then the functional $\mathcal{L}$ is of class C^1 on

$$D((-\Delta)^\alpha) \times D((-\Delta)^{1-\alpha}), \tag{1.7}$$

for some $\alpha \in (0,1)$, depending on p and q, where $(-\Delta)^\alpha$ is the fractional power of the selfadjoint operator $-\Delta$ with domain $W^{2,2} \cap W_0^{1,2}(\Omega) \subset L^2(\Omega)$. The shortcoming of this approach is the fact that one is not able to formulate the problem variationally

Received for publication May 1994.
This work was reported on at the International Conference on Differential Equations in August, 1993.
AMS Subject Classifications: 35J50, 35J55, 46D05, 47H05, 49A27, 49A51, 49A55, 49B40.