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ANISOTROPIC EQUATIONS IN L^1

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1. Introduction. Existence results for weak solutions of equations involving the p-Laplacian with right hand side measure have been recently obtained in [1]. G. Stampacchia [5] first considered the problem in the linear case; he obtained an existence result through a *duality method*: if $T \in W^{-1,r}(\Omega)$, $r > N$, then the solution of the Dirichlet problem

$$w \in H_0^1(\Omega) : -\operatorname{div}(A(x)Dw) = T \tag{1.1}$$

lies in $C^0(\Omega)$ and the mapping $T \to w$ is linear and continuous from $W^{-1,r}(\Omega)$ to $C^0(\Omega)$. Therefore the adjoint operator maps $M_b(\Omega)$ into $W_0^{1,q}(\Omega)$, for any $q < \frac{N}{N-1}$.

In this paper we consider anisotropic equations with right hand side measures. We confine ourselves to the model case

$$\begin{cases} -\operatorname{div}(j(Du)) = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

where $j(\xi)$ is the vector field whose components are $|\xi_i|^{p_i-2}\xi_i (i = 1, \dots, N; p_i > 1)$. We shall prove the existence of solutions of (1.2). More precisely we obtain the existence of a solution in the Sobolev space

$$W_0^{1,q_i}(\Omega)\big\{v \in W_0^{1,1}(\Omega) : \frac{\partial v}{\partial x_i} \in L^{q_i}(\Omega), \quad \forall i = 1, \dots, N\big\},$$

where q_i (for $i = 1, \dots, N$) is any real number greater than 1 and such that

$$1 < q_i < \frac{N(\bar{p}-1)}{\bar{p}(N-1)}\, p_i,$$

and

$$\frac{1}{\bar{p}} = \frac{1}{N}\sum_{i=1}^{N} \frac{1}{p_i}, \quad p_i > 1. \tag{1.3}$$

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