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# ANDERSON MODEL WITH LÉVY POTENTIAL

John M. Noble
Department of Statistics, University College Cork, Ireland

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**1. Introduction.** The purpose of this article is to extend the results of the paper "Anderson Model with Gaussian Potential"([2]). In particular, we want to extend discussion of existence and uniqueness of the equation discussed there to the case where the potential $\zeta$ is a family of semimartingales and the motion process is possibly time inhomogeneous with right-continuous sample paths. Existence and uniqueness for Lévy potential of bounded variation, where the motion process is a Poisson process on $\mathbb{Z}^d$, has already been considered by Ahn-Carmona-Molchanov in [1]. Importantly, they analyzed the large time asymptotics of the moments of the solution. However, it is particularly important that we produce Doléans-style Kac representation for the solution. These generalizations for our model seemed necessary when it transpired from conversations that our model was closely connected with certain problems in Polymer Physics. The connection with the polymer model is treated in the paper [3], where the Doléans representation is seen to be correct for producing the partition function. To be more precise, let $(\Omega, \mathcal{G}, (\mathcal{G}_s)_{s\geq 0}, \mathbb{Q})$ be a filtered probability space. Let $(\mathbb{E}, \mathcal{E})$ be a measurable space and let

$$\zeta : \mathbb{R}_+ \times \mathbb{E} \times \Omega \longrightarrow \mathbb{R}$$

be a family of semimartingales under the usual conditions with joint quadratic variation structure given by

$$[\zeta(x), \zeta(y)]_t = \Lambda(x, y, t, \omega).$$

For fixed $(x, y) \in \mathbb{E} \times \mathbb{E}$, $\Lambda$ is of integrable variation in time. The precise hypotheses are given in the text. We discuss the stochastic partial differential equation

$$du_t = L_t u_{t-} dt + u_{t-} d\zeta_t, \quad u_0 \equiv \phi,$$

where $(L_t)_{t>0}$ is the infinitesimal generator of a time inhomogeneous Markov process, with state space $(\mathbb{E}, \mathcal{E})$ with right-continuous sample paths and with infinite life time. When $\zeta$ is a process of square summable jumps, then the solution for this equation is given by

$$u_t(x) = E_x[\phi(w_{0,t}) \prod_{0<s\leq t} (1 + \Delta\zeta_s(w_{s,t}))],$$

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