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A NONRESONANCE RESULT WITH RESPECT TO THE FUČIK SPECTRUM FOR A SECOND ORDER DIFFERENTIAL EQUATION

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1. Introduction. This paper deals with the existence of solutions to the following problem

$$\begin{cases} -u''(t) = f(t, u(t)), \quad t \in [0, 2\pi] \\ u(0) = u(2\pi); \quad u'(0) = u'(2\pi). \end{cases} \tag{1.1}$$

We are interested in cases where $f : [0, 2\pi] \times \mathbb{R} \to \mathbb{R}$ is asymptotically linear and "asymmetric." By this we mean that the behaviour of $\frac{f(t,s)}{s}$ for s going to $+\infty$ may be different from what it is for s going to $-\infty$ ("jumping" nonlinearities).

Fučik and Dancer [1, 2], who first studied this problem, showed that the solvability of (1.1) strongly depends on the behaviour of $\lim_{s\to+\infty} \frac{f(t,s)}{s}$, $\lim_{s\to-\infty} \frac{f(t,s)}{s}$ with respect to some set $\Sigma \subset \mathbb{R}^2$. This set, the so-called Fučik spectrum, is the set of $(\alpha, \beta) \in \mathbb{R}^2$ for which the problem (1.1) with $f(t, s) = \alpha s_+ - \beta s_-$ has a nontrivial solution ($s_+ = \max\{s, 0\}$, $s_- = (-s)_+$). It is well known and it can be easily verified that Σ is composed of two lines $\mathbb{R} \times \{0\}$, $\{0\} \times \mathbb{R}$ and the curves C_N, $N \geq 1$

$$C_N = \Big\{(\alpha, \beta) \in \mathbb{R}^+_* \times \mathbb{R}^+_*;\ \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{2}{N}\Big\}. \tag{1.2}$$

Subsequently, various conditions have been introduced in order to guarantee nonresonance in problem 1.1 (cf. [3]–[6]). In particular, in [6], Gossez-Omari used the following assumptions:

$$\begin{cases} \text{There exist two points } (\alpha, \beta) \in C_N, (a, b) \in C_{N+1} \text{ for some} \\ N \geq 0 \text{ (here } C_0 = \{(0, 0)\}\text{) such that} \\ \alpha \leq \lambda(t) = \liminf\limits_{s\to+\infty} \dfrac{f(t, s)}{s} \leq \limsup\limits_{s\to+\infty} \dfrac{f(t, s)}{s} = p(t) \leq a \\ \beta \leq \mu(t) = \liminf\limits_{s\to-\infty} \dfrac{f(t, s)}{s} \leq \limsup\limits_{s\to-\infty} \dfrac{f(t, s)}{s} = q(t) \leq b \end{cases} \tag{1.3}$$

and

$$\begin{cases} \text{the sets } \{t \in [0, 2\pi]; \lambda(t) > \alpha, \mu(t) > \beta\} \text{ and} \\ \{t \in [0, 2\pi]; p(t) < a, q(t) < b\} \quad \text{have positive measure.} \end{cases} \tag{1.4}$$

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