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DISCONTINUOUS SEMILINEAR PROBLEMS IN VECTOR-VALUED FUNCTION SPACES

D. Motreanu
Universitatea Al.I.Cuza, Seminarul Matematic Al.Myller, 6600 Iaşi, Romania

Z. Naniewicz
Institute of Applied Mathematics and Mechanics, University of Warsaw, 02-097 Warsaw, Poland

(Submitted by: Viorel Barbu)

1. Introduction. The aim of this paper is to study the existence of solutions to the following discontinuous semilinear problem:

Problem (P): Find $u \in V$ such that

$$a(u,v) - \langle f, v\rangle_V + \int_\Omega j^0(u,v)\, d\Omega \geq 0, \quad \forall v \in V, \tag{1.1}$$

where V is a reflexive Banach space compactly imbedded into $L^p(\Omega; R^N)$ with $p > 2$ and $N \geq 1$, for a bounded domain Ω in R^m, $m \geq 1$. Throughout Section 2 it will be supposed that $V \cap L^\infty(\Omega, R^N)$ is dense in V. We use the symbols V^*, $\langle\cdot,\cdot\rangle$, $\|\cdot\|_V$, $\|\cdot\|_{L^p(\Omega)}$ to denote the dual space of V, the pairing over $V^* \times V$, the norm in V and the norm in $L^p(\Omega; R^N)$, respectively. The data entering (1.1) are the following: $a(\cdot,\cdot)$ is a continuous symmetric bilinear form on V satisfying the ellipticity condition:

$$a(v,v) \geq \alpha\|v\|_V^2, \quad \forall v \in V,\ \alpha = const. > 0, \tag{1.2}$$

with the associated operator $A : V \to V^*$ defined by

$$a(v,u) = \langle Av, u\rangle_V, \quad \forall u, v \in V,$$

$f \in V^*$ and $j : R^N \to R$ is a locally Lipschitz function. The notation $j^0(\cdot,\cdot)$ stands for the Clarke's directional differential defined by

$$j^0(\xi,\eta) = \limsup_{\substack{h\to 0\\ \lambda\downarrow 0}} \lambda^{-1}(j(\xi+h+\lambda\eta) - j(\xi+h)), \quad \forall \xi,\eta \in R^N$$

(cf. [10]) by means of which the Clarke's generalized gradient is introduced as

$$\partial j(\xi) = \{\eta \in R^N : j^0(\xi,y) \geq \langle \eta, y\rangle_{R^N}, \quad \forall y \in R^N\} \tag{1.3}$$

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