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NONEXISTENCE OF POSITIVE SOLUTIONS OF SEMILINEAR ELLIPTIC SYSTEMS IN $\mathbf{R}^N$

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1. Introduction. In this paper we study nonexistence of positive classical solutions of elliptic systems of the form

$$-\Delta u = f(u,v), \quad -\Delta v = g(u,v) \quad \text{in } \mathbb{R}^N, \tag{1.1}$$

where $N \geq 3$ and f, g are given nonnegative functions such that $f(0,0) = g(0,0) = 0$.

Recently the existence and nonexistence of positive solutions for the scalar equation

$$-\Delta u = f(x,u) \quad \text{in } \mathbb{R}^N, \tag{1.2}$$

has been studied by a number of authors; see for example [2, 4, 5, 9, 10, 13, 14, 16] and the references therein. In particular in Gidas-Spruck ([5]) it is proved among other things that if $f(x,u) = |u|^{p-1}u$ then (1.2) has no positive classical solutions provided $1 < p < (N+2)/(N-2)$. An important feature of their result is that no a priori information on the behavior of the solutions at infinity is given.

Liouville-type theorems of this kind play an important role in the study of semilinear elliptic and parabolic equations as is shown in [6, 7, 8]. Hence it is a natural question, in view of the possible applications, to investigate when similar results continue to hold for systems of equations. Some interesting nonexistence results have for example been obtained for systems of semilinear elliptic equations of potential type by Noussair and Swanson ([17]).

The paper is organized as follows. In Section 2 we prove some Liouville theorems of Gidas-Spruck type for systems of the form (1.1) and for related equations. In Section 3 we consider nonexistence of positive radially symmetric solutions for a nonautonomous system and for a related nonvariational problem (see Theorem 3.3). Finally in Section 4 we discuss some results for the Dirichlet problem in a ball.

2. Non-existence results. Let $u \in C^2(\mathbb{R}^N)$, $(N \geq 3)$ and consider its average on a sphere of radius $r = |x|$ centered at the origin; that is,

$$T(u)(x) = u^{\#}(x) = |S_{N-1}|^{-1} \int_{S_{N-1}} u(r\omega)\, d\omega,$$

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