*Differential and Integral Equations* Volume 9, Number 4, July 1996, pp. 635 – 653

# NON-EXISTENCE OF POSITIVE SOLUTIONS OF LANE-EMDEN SYSTEMS

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**1. Introduction.** In this paper we consider (component-wise) positive solutions of the weakly coupled system

$$\begin{aligned} \Delta u + v^p &= 0, \\ \Delta v + u^q &= 0, \end{aligned} \qquad x \in \mathbb{R}^n, \tag{I}$$

where $p, q > 0$ and $n \geq 3$ is the dimension of the space, and are concerned with the question of non-existence of such solutions. This system arises in chemical, biological and physical studies, and has been investigated by several authors, see for example [3, 8, 9] and references therein.

The system (I) is a natural extension of the celebrated Lane-Emden equation, and we thus refer to it as the Lane–Emden system. The Lane-Emden equation

$$\Delta u + u^p = 0, \qquad x \in \mathbb{R}^n,\ n > 2,\ p > 1 \tag{II}$$

has been extensively studied, going back to the pioneering work of Fowler (cf. [4], and the recent paper [12]). It is well-known that the Sobolev exponent

$$l = \frac{n+2}{n-2}$$

serves as the dividing number for existence and non-existence of solutions of (II), that is, equation (II) admits non-negative, non-trivial solutions if and only if $p \geq l$, see [4] and [5].

It is natural to ask if there exists a corresponding *dividing curve* in the $pq-$plane for the Lane-Emden system, that is, a curve with the property that (I) admits positive solutions if and only if $(p, q)$ is on or above the curve.

Mitidieri [9] showed that (I) does not have any positive *radial* solutions if

$$\frac{1}{p+1} + \frac{1}{q+1} > \frac{n-2}{n}, \qquad p,\ q > 1,$$

Received for publication February 1996.
*Research supported in part by NSF grant DMS-9418779, by a grant from Alabama EPSCoR and a faculty research grant from University of Alabama at Birmingham, Birmingham.
AMS Subject Classifications: 35J60, 35B40.