RADIAL SOLUTIONS FOR A NONLINEAR PROBLEM WITH p–LAPLACIAN

G. Dincă
Faculty of Mathematics, St. Academiei 14, 70109 Bucharest, Romania

P. Jebelean
Faculty of Mathematics, Bv. V. Pârvan 4, 1900 Timişoara, Romania

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1. Introduction. In paper [2] the author studies the existence of positive radial solutions for the Dirichlet problem attached to a semilinear elliptic equation of type

$$\triangle u = g(|x|, u) \quad \text{in} \quad \mathcal{C}(a, b),$$

where $0 < a < b < \infty$ and $\mathcal{C}(a, b)$ is the annulus $\{x \in \mathbf{R}^n : a < |x| < b\}$, $n \geq 2$. As usual the above problem is reduced to a differential one. The approach is based on the Mountain Pass Theorem.

The case $a = 0$ leads to a singular problem. This case agrees with the Dirichlet problem in a ball; the condition $u(0) = 0$ changes into $\frac{du}{d\mathbf{r}}(0) = 0$, where $\frac{du}{d\mathbf{r}}$ stands for the radial derivative. Moreover, the classical Laplacian is a particular case of the p-Laplacian ([1])

$$\triangle_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$$

with $1 < p < \infty$, corresponding to $p = 2$. The above remarks guided us to consider radial solutions for the nonlinear problem with p-Laplacian

$$\text{(L)} \quad \begin{cases} \triangle_p u = g(|x|, u, \frac{du}{d\mathbf{r}}) & \text{in } \Omega \\ u|_{|x|=1} = 0, \quad \lim_{x \to 0} \frac{du}{d\mathbf{r}}(x) = 0, \end{cases}$$

where $\Omega = \{x \in \mathbb{R}^n, 0 < |x| \leq 1\}$.

In trying to find radial solutions for problem (L), we are led to consider the singular boundary value problem

$$(r - \text{L}) \quad \begin{cases} (r^{n-1}|v'(r)|^{p-2} v'(r))' = r^{n-1} g(r, v(r), v'(r)) & \text{in } T \\ v(1) = 0, \quad \lim_{r \to 0} v'(r) = 0, \end{cases}$$

where $T = (0, 1]$.

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