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PERIODIC SOLUTIONS FOR A CLASS OF NONAUTONOMOUS WAVE EQUATIONS

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1. Introduction. In this paper we will be concerned with the existence of nontrivial solutions for the superlinear wave equation

$$\Box u := u_{tt} - u_{xx} = bu + g(t,x,u), \qquad t \in \mathbf{R},\ 0 < x < \pi, \tag{1.1}$$

satisfying the boundary and the periodicity conditions

$$\begin{cases} u(t,0) = u(t,\pi) = 0, & t \in \mathbf{R}, \\ u(t+2\pi,x) = u(t,x), & t \in \mathbf{R},\ 0 < x < \pi. \end{cases} \tag{1.2}$$

Here g is continuous, 2π-periodic in t, superlinear at 0 and $\pm\infty$, and the mapping $\xi \mapsto f(t,x,\xi) := b\xi + g(t,x,\xi)$ is increasing. In [7] Rabinowitz showed that (1.1), (1.2) has a nontrivial solution if $b = 0$ and $g = g(\xi)$. The same result was shown to hold if $b \geq 0$ and $\xi \mapsto g(t,x,\xi)$ is increasing. If $f = f(x,\xi)$, a much stronger result was obtained in [8], where it was shown that (1.1), (1.2) has solutions of arbitrarily large L^∞-norm under the conditions that f is strictly increasing and superlinear at $\pm\infty$ (cf. also Tanaka, [9]). The proofs in [7, 8] are rather complicated. In [2], see also [1], Brézis, Coron and Nirenberg gave a simpler proof of the result in [7] for $b = 0$ and $g = g(\xi)$. Another simpler proof (again for $b = 0$ and $g = g(\xi)$, but admitting discontinuous g) was given in [4]. A simpler proof of the result in [8] may be found in [3].

In this paper we generalize Rabinowitz's result ([7]). Recall that

$$f(t,x,\xi) = b\xi + g(t,x,\xi)$$

and suppose f satisfies the following conditions:

(f_0) $f \in C(\mathbf{R} \times [0,\pi] \times \mathbf{R}, \mathbf{R})$ is 2π-periodic in t and there exist $s > 2$, $a_1, a_2 > 0$ such that

$$|f(t,x,\xi)| \leq a_1 + a_2|\xi|^{s-1} \quad \text{for all} \quad t,x,\xi;$$

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