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# LARGE-TIME BEHAVIOR OF SOLUTIONS OF A SYSTEM OF PDE'S GOVERNING DIFFUSION PROCESSES IN A HETEROGENEOUS MEDIUM*

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(Submitted by: J.A. Goldstein)

We study the large-time behavior of solutions of the initial-boundary-value problem for a system of nonlinear partial differential equations of the form

$$\begin{aligned} \frac{\partial u_1}{\partial t} - \Delta\phi_1(u_1) + \gamma(\phi_1(u_1) - \phi_2(u_2)) &\ni 0 && \text{in} \quad \Omega \times (0,\infty) \\ \frac{\partial u_2}{\partial t} - \Delta\phi_2(u_2) - \gamma(\phi_1(u_1) - \phi_2(u_2)) &\ni 0 && \text{in} \quad \Omega \times (0,\infty) \\ -\frac{\partial\phi_1(u_1)}{\partial\eta} &\in \beta_1(u_1) && \text{on} \quad \partial\Omega \times (0,\infty) \\ -\frac{\partial\phi_2(u_2)}{\partial\eta} &\in \beta_2(u_2) && \text{on} \quad \partial\Omega \times (0,\infty) \\ (u_1(0), u_2(0)) &= (u_{01}, u_{02}) && \text{in} \quad \Omega, \end{aligned} \tag{I}$$

where $\Omega$ is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $\partial/\partial\eta$ denotes the Neumann boundary operator, $\beta_i$, $\phi_i$ and $\gamma$ are maximal monotone graphs in $\mathbb{R}\times\mathbb{R}$ with $0 \in \beta_i(0)$, $0 \in \phi_i(0)$ and $0 \in \gamma(0)$. A particular case of system (I) is proposed by E. DiBenedetto and R.E. Showalter in [12] as a mathematical model for heat conduction in a composite material consisting of two components and under the assumption that the first component occurs in small isolated parts that are suspended in the second component, which implies the change of phase occurs in the second component. In this situation $u_1$ and $u_2$ represent the temperatures in the first and second components, respectively, $\phi_1 \equiv 0$ and $\phi_2(s) = bs + LH(s)$ where $b > 0$, $L > 0$ and $H$ is the multivalued Heaviside step function. Based on the physical analysis in [12], one may still view (I) as a mathematical description of diffusion processes within a medium composed of two components which involves phase change. In this connection, $\gamma$ is related to the surface area common to the two components. Thus, $\gamma$ is a measure of the homogeneity of the material.

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