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PARABOLIC VARIATIONAL INEQUALITIES WITH SINGULAR INPUTS

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1. Introduction. This work is concerned with abstract parabolic variational inequalities of the form

$$\begin{aligned} &dy(t) + Ay(t)dt + \partial\varphi(y(t))dt \ni f(t)dt + dM(t), \quad t \in [0,T], \\ &y(0) = y_0, \end{aligned} \tag{1.1}$$

in a real Hilbert space H, where A is a linear self–adjoint operator, $\partial\varphi$ is the subdifferential of a convex lower semicontinuous function $\varphi : H \to \overline{\mathbb{R}}$ and M is a continuous function from $[0,T]$ to H or to a smaller space.

Equation (1.1) is considered in the weak sense

$$y(t) + \int_0^t Ay(s)\,ds + \eta(t) = y_0 + \int_0^t f(s)\,ds + M(t), \quad \forall\, t \in [0,T], \tag{1.2}$$

where η is a function with bounded variation on $[0,T]$ such that

$$d\eta(t) \in \partial\varphi(y(t))dt$$

in some weak sense.

The first motivation to study such a problem comes from the classical Skorohod problem:

Given $M \in C([0,T];\mathbb{R}^d)$, $M(0) = 0$ and $x \in D \subset \mathbb{R}^d$, D a convex set, find $y \in C([0,T];\mathbb{R}^d)$ and $\eta \in C([0,T];\mathbb{R}^d) \cap BV([0,T];\mathbb{R}^d)$, $\eta(0) = 0$ such that

$$y(t) + \eta(t) = x + M(t), \quad \forall\, t \in [0,T], \tag{1.3}$$

$$\int_0^T 1_{\mathrm{int}\,D}(y(s))\,d[\eta]_s = 0, \tag{1.4}$$

$$\eta(t) = \int_0^t n_s\,d[\eta]_s, \quad \forall\, t \in [0,T], \tag{1.5}$$

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