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PERIODIC MOTIONS OF A LATTICE OF PARTICLES WITH SINGULAR FORCES

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Abstract. We prove the existence of periodic motions of prescribed energy E or period T for an autonomous Hamiltonian system consisting of a lattice of N particles with one fixed endpoint. Each interparticle force is defined over a bounded interval of the real line, the potentials of such forces having a barrier at the endpoints of the interval; a physical model for this system is suggested. The proofs are based on variational methods and by suitable sequences of approximating problems that involve strong-force terms: this technique also allows one to obtain the existence of periodic bounce trajectories in a rectangular billiard.

1. Introduction. In this paper we consider an autonomous Hamiltonian system consisting of a lattice of N ($N \geq 2$) particles; the state of the lattice at time t is represented by a vector $q(t) = \{q_1(t), \ldots, q_N(t)\}$, where $q_i(t)$ represents the state of the i-th particle; we study lattices with one fixed endpoint, i.e., $q_0 \equiv 0$. In our system each particle of the lattice interacts only with the nearest neighbors; denote by V_{i+1} the potential of the interaction between the i-th and the $(i+1)$-th particle, then if such potentials are smooth the equations governing the motion of the lattice are

$$\begin{aligned} \ddot{q}_i(t) &= V_i'[q_{i-1}(t) - q_i(t)] - V_{i+1}'[q_i(t) - q_{i+1}(t)] \qquad i = 1, \ldots, N-1 \\ \ddot{q}_N(t) &= V_N'[q_{N-1}(t) - q_N(t)] \; . \end{aligned} \tag{1}$$

The starting point of studies of motions in lattices is the celebrated numerical experiment of Fermi-Pasta-Ulam ([17]); their model represents a chain of disks which are allowed to rotate around their axes, the variables $q_i(t)$ being the values of the angle of rotation; the nearest neighbor disks are connected with a spring which raises a force, which is linear plus a perturbing term. They observed that if the perturbation is small enough, then the system does not converge to equipartition of energy. The idea that a strongly nonlinear dynamical system could lack ergodicity was supported by the existence of periodic motions for lattices of particles with exponential interactions proved by Toda ([25]); in that case the system is completely integrable and admits explicit periodic and soliton solutions. Later, this problem has been studied by variational methods for a class of large perturbations of the Toda interactions and

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