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SUFFICIENT CONDITIONS FOR MINIMA OF SOME TRANSLATION INVARIANT FUNCTIONALS

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I. Introduction. We consider the question of minimizing certain functionals subject to a constraint. For the problems we are going to study, a very powerful method, concentration-compactness, introduced by P.L. Lions ([1]), has been developed. This method consists in showing that strict subadditivity is a necessary and sufficient condition for precompactness of minimizing sequences; of course the strict subadditivity has to be verified and this has been done for many important problems in both Pure and Applied Mathematics.

In this paper we present a slightly different point of view. What we do is to replace strict subadditivity by the verification of the assumption H_4 in part II or H_5 in part III. Our results are just sufficient for the convergence of minimizing sequences and a motivation for them is that they cover some examples for which the verification of the strict subadditivity, apparently, would not be easy. Of course we can not say that it would be impossible because it is a necessary condition. In [2] we have already used this method in a simpler problem and here we consider functionals with nonlocal terms and functionals with two dependent variables (giving rise to elliptic systems). All problems we have treated have just one constraint; in the case of more than one (as in [6]) it seems difficult to use our method.

There is a very large literature about the topic we treat but our list of references contains only the papers which are more closely related to ours.

II. Functionals with nonlocal terms. In this section we consider the question of minimizing the functional

$$V(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\operatorname{grad} u(x)|^2\,dx + \frac{1}{2}\int_{\mathbb{R}^N\times\mathbb{R}^N} k(x-y)H(u(x))H(u(y))\,dx\,dy + \int_{\mathbb{R}^N} F(u(x))\,dx \tag{II.1}$$

in the set of nonnegative elements u in $H^1(\mathbb{R}^N)$ such that

$$I(u) = \int_{\mathbb{R}^N} u^2(x)\,dx = \lambda > 0. \tag{II.2}$$

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