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MULTIPLE SOLUTIONS OF NONHOMOGENEOUS ELLIPTIC EQUATION WITH CRITICAL NONLINEARITY

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1. Introduction. The main purpose of this work is to investigate the existence of multiple positive solutions of the following problem:

$$\begin{cases} -\Delta u = Q(x)|u|^{p-2}u + \epsilon h(x) \text{ in } \Omega \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1_n}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$ $(N \geq 3)$, $p = \frac{2N}{N-2}$ is a critical Sobolev exponent, $h \in L^2(\Omega)$, with $h \geq 0$, $\not\equiv 0$ on Ω, $Q \in C(\bar{\Omega})$ is positive and $\epsilon > 0$ is a parameter.

In recent years several authors have studied problems of this nature (see for example [3], [4], [16], [14], [15]. In particular, in the case where $Q(x) \equiv 1$ on Ω, Tarantello ([17]) proved the existence of at least two positive distinct solutions for $\epsilon > 0$ small. This result has been extended by Rey ([16]) who proved that problem (1_n) has at least $\text{cat}\ \Omega + 1$ positive distinct solutions for $\epsilon > 0$ small.

In this paper we are concerned with the effect of the shape of the graph of Q on the number of positive solutions. Throughout this paper we assume the hypothesis

(Q) $Q \in C(\bar{\Omega})$, $Q > 0$ on $\bar{\Omega}$ and there exist points $a_1, \ldots, a_k \in \Omega$ where Q takes on strict local maxima; i.e., $Q(a_j) = \max_{x\in\Omega} Q(x)$ and $Q(x) < Q(a_j)$ for x in a neighbourhood U_j of a_j, $j = 1, \ldots, k$, and moreover for $x \in U_j$

$$Q(x) - Q(a_j) = o\big(|x - a_j|^{\frac{N-2}{2}}\big).$$

In what follows we use the notation $Q_M = \max_{x\in\Omega} Q(x)$. The main results of this paper are the following:

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