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ON SELF-SIMILAR SOLUTIONS OF THE THIN FILM EQUATION WHEN $n = 3$

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1. Introduction. The motion of thin films in which the surface tension at the air/liquid interface determines the flow, is frequently described by the *Thin Film equation*

$$h_t + (h^n h_{xxx})_x = 0, \quad h \geq 0, \quad x \in \mathbf{R},\ t > 0, \tag{1.1}$$

in which h denotes the thickness of the film, t the time, x the distance along the film, and n is a *positive* constant, which depends on the type of flow considered.

In a Hele-Shaw flow between two parallel plates, the flow in a thin film spanned between the plates, separated by a distance b, is governed by the equations:

$$h_t + (hu)_x = 0 \qquad \text{Conservation of mass,} \tag{1.2}$$

$$u = -\frac{b^2}{12\mu} p_x \qquad \text{Darcy's Law} \tag{1.3}$$

Here u denotes the velocity of the fluid in the film, averaged over the thickness of the film, μ the viscosity of the fluid and p_x the pressure gradient along the film. If the flow is determined by the surface tension we may set to first approximation

$$p_x = -\gamma\kappa = -\gamma h_{xx}, \tag{1.4}$$

where γ denotes the surface tension and κ the curvature of the surface. When we substitute (1.4) into (1.3), and the result into (1.2), we obtain

$$h_t + \frac{b^2\gamma}{12\mu}(h\, h_{xxx})_x = 0, \tag{1.5}$$

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