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SOME CHARACTERIZATIONS OF THE TAYLOR–COUETTE ATTRACTOR

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Abstract. The Taylor–Couette problem in infinite cylinders is considered for weakly unstable Couette flow in case of fixed outer cylinder. The following results about the attractor are established in this paper: 1) The upper–semicontinuity of the rescaled Taylor–Couette attractor towards the associated Ginzburg–Landau attractor holds. 2) Every solution in the Taylor–Couette attractor can be shadowed by some pseudo–orbit in this Ginzburg–Landau attractor. 3) The Taylor–Couette attractor only contains rotational symmetric solutions. Similar results hold for corresponding hydrodynamical stability problems, like Bénard's problem.

1. Introduction. In this paper we prove some results about the Taylor-Couette attractor in infinite cylinders in case of weakly unstable Couette flow. Similar results hold for corresponding hydrodynamical stability problems, like Bénard's problem, or reaction–diffusion systems, as the Brusselator, and many other systems.

The Taylor–Couette problem is a classical hydrodynamical stability problem, where a fluid is contained between two rotating concentric cylinders. Originally designed for the study of turbulence, it is nowadays a well studied problem in pattern formation. This problem is modeled with infinite cylinders in order to neglect the boundaries at the top and at the bottom of the cylinders, and to obtain the main issues of the pattern forming processes, i.e., to obtain a dynamics independent of the special length of the cylinders.

The Taylor–Couette problem with periodic boundary conditions can be studied with classical bifurcation analysis. An almost complete overview about this case can be found in the textbook [3] to which we refer for additional information. Here we consider the situation where the trivial ground

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