Differential and Integral Equations Volume 13 (1–3) January–March 2000, pp. 217–226

ON THE ANTIMAXIMUM PRINCIPLE AND THE FUČIK SPECTRUM FOR THE NEUMANN P-LAPLACIAN *

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(Submitted by: Klaus Schmitt)

1. Introduction. Let us start by considering the linear Neumann problem

$$Lu = \lambda u + h(x) \ \text{ in } \Omega, \quad \partial u/\partial \nu_L = 0 \ \text{ on } \partial\Omega. \tag{1.1}$$

Here Ω is a smooth bounded domain in $\mathbb{R}^N$, L is a second order linear symmetric elliptic operator in divergence form on Ω and $\partial/\partial\nu_L$ denotes the associated conormal derivation on $\partial\Omega$. Let μ_1 be the first eigenvalue of L under the above boundary conditions.

In this context the antimaximum principle (cf. [2]) asserts that given $h \geq 0$, $h \not\equiv 0$, there exists $\delta = \delta(h) > 0$ such that if $\lambda \in]\mu_1, \mu_1 + \delta[$, then any solution u of (1.1) satisfies $u < 0$ on $\bar{\Omega}$. Moreover when $N = 1$, δ can be taken independent of h. We say in this latter case that the antimaximum principle holds *uniformly* and we denote by δ_1 the largest δ admissible.

As observed in [4], there exists a connection between the antimaximum principle and the behaviour at infinity of the corresponding Fučik spectrum. We recall that this spectrum is defined as the set Θ of those $(\alpha, \beta) \in \mathbb{R}^2$ such that

$$Lu = \alpha u^+ - \beta u^- \ \text{ in } \Omega, \qquad \partial u/\partial \nu_L = 0 \ \text{ on } \partial\Omega \tag{1.2}$$

*Partly supported by the European project ERBCHRXCT940555.
†Partly supported by DGES PB95-1203, M. E. C., Spain.
Accepted for publication June 1998.
AMS Subject Classifications: 35P30, 35J20.