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EXISTENCE THEOREM OF PERIODICAL SOLUTIONS OF HAMILTONIAN SYSTEMS IN INFINITE-DIMENSIONAL HILBERT SPACES

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1. Introduction. The aim of this paper is to state and prove an existence result of periodical solutions for abstract Hamiltonian systems

$$\mathcal{A}u(t) = \nabla H(t, u(t)), \qquad u(0) = u(T) \tag{1}$$

where X is a Hilbert space, $\tilde{X} = X \times X$ and $H : [0,T] \times \tilde{X} \to \mathbf{R}$ is measurable in t for each $u \in \tilde{X}$ and continuously differentiable and convex in u for almost every $t \in [0,T]$, and $A : D(A) \subset X \to X$ is such that $-A$ generates a C_0 semigroup e^{-At} on the space X and e^{-AT} is compact and

$$\mathcal{A} : D(\mathcal{A}) \subset \tilde{X} \leftarrow \tilde{X}, \quad \mathcal{A}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} A^*p - \dot{p} \\ Aq + \dot{q} \end{pmatrix}.$$

This type of system was studied for the first time by Barbu [1], in a more general case. We will use the duality theory developed for Hamiltonian systems defined on finite-dimensional spaces by Mawhin and Willem in [6] and [7].

2. Preliminary results. We develop and prove in this section some preliminary tools of general character. All these facts will be used in proofs of our main result. For more details see [2] or [3].

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