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THE CAUCHY PROBLEM FOR THE KADOMTSEV-PETVIASHVILI EQUATION (KPII) IN SOBOLEV SPACES H^S, $S > 0$*

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0. Introduction. In this article we study the boundedness properties of a bilinear form associated with the Kadomtsev-Petviashvili equation (KPII), in order to establish that the Cauchy problem for this equation,

$$(u_t + u_{xxx} + uu_x)_x + u_{yy} = 0, \quad u(x, y, 0) = u_0(x, y), \tag{0.1}$$

is well posed for initial datum u_0 in the Sobolev spaces H^s with $s > 0$.

The IVP (0.1) belongs to a wide class of nonlinear dispersive evolution problems which can be written in the form

$$\partial_t u - im(D)u = F(u), \quad u(x, 0) = u_0(x), \tag{0.2}$$

where $u = u(x, t)$, $x \in \mathbb{R}^n$, $t > 0$, F is a nonlinear operator and $m(D)$ is a linear operator defined through the Fourier transform in the space variables by $[m(D)f]\widehat{\ }(\zeta) = m(\zeta)\widehat{f}(\zeta)$, with $m(\zeta)$ being a real-valued rational function on $\zeta \in \mathbb{R}^n$.

In fact, problem (0.1) is obtained from (0.2) by taking $n = 2$, $\zeta = (\xi, \eta)$, $F(u) = -\frac{1}{2}\partial_x(u^2)$, $m(\zeta) = \xi^3 - \eta^2/\xi$. Equation KPI is the one for which $m(\zeta) = \xi^3 + \eta^2/\xi$.

If $W(t)u_0$ is the solution at t of the linear problem associated with (0.2), then problem (0.2) can be formulated by means of the equivalent integral equation

$$u(t) = W(t)u_0 + \int_0^t W(t - t')F(u(t'))\,dt' \ . \tag{0.3}$$

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