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ANISOTROPIC ESTIMATES AND STRONG SOLUTIONS OF THE PRIMITIVE EQUATIONS

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1. Introduction

A large number of geophysical fluids are modeled by the so-called "Primitive Equations." This model is obtained formally from the Navier–Stokes equations, with anisotropic (eddy) viscosity, assuming two important simplifications: "hydrostatic pressure" (depending linearly on the depth) and "rigid lid hypothesis" (fixed water surface); see [10], [14] and references therein cited.

The model. For simplicity, we take constant density and assume that the effects due to the temperature (and salinity) can be decoupled from the dynamic of the flow. Then, we have a three-dimensional flow induced by the wind tension on the surface and by the centripetal and Coriolis forces. When the Earth's curvature is not considered, we can use Cartesian coordinates instead of spherical coordinates (see in Lions–Temam–Wang [12] the model with spherical coordinates); hence, the Lipschitz-continuous domain Ω is given by

$$\Omega = \{(\vec{x}, z) \in \mathbb{R}^3;\, \vec{x} \in \omega, -D(\vec{x}) < z < 0\}, \tag{1}$$

where $\omega \subseteq \mathbb{R}^2$ is an open domain and $D : \overline{\omega} \to \mathbb{R}_+$ is the depth function. The different boundaries of Ω (surface, bottom and sidewalls) are respectively

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