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SOME SPECIAL SOLUTIONS OF THE THIN-FILM EQUATION

Elena Beretta
Dipartimento di Matematica "G. Castelnuovo," Università di Roma
"La Sapienza," P.le A. Moro 5, 00185 Roma, Italy

Michiel Bertsch
Dipartimento di Matematica, Università di Roma "Tor Vergata"
Via della Ricerca Scientifica, 00133 Roma, Italy

(Submitted by: L.A. Peletier)

1. Introduction

The one-dimensional spreading of a thin liquid film over a horizontal solid surface is often modelled by the following equation for the film height h:

$$\partial_t h + \partial_x \left(h^3 \partial^3_{xxx} h \right) = 0. \tag{1.1}$$

Here $h(x,t)$ is a nonnegative function which typically has compact support. The derivation of equation (1.1) is based on the following basic assumptions: the lubrication approximation with the no-slip condition for the fluid at the solid surface and the fact that the pressure is entirely created by surface tension. We observe that, if the lubrication approximation is assumed to be valid up to the edge of the support of the film (the contact lines), then the height h will satisfy a zero-flux condition at the contact lines. The main difficulty of the lubrication model is that the zero-flux condition implies the nonexistence of solutions of (1.1) with *advancing* contact lines. In this context in [3], Bernis, Peletier and Williams prove the nonexistence of compactly supported source-type solutions of self-similar form, and in [4], Boatto, Kadanoff and Olla prove the nonexistence of advancing travelling waves.

In the paper [1] we introduced a new mathematical model based on the lubrication approximation in the major part of the film where the film is not too thin (basic region), and in the remaining small regions near the contact

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