Differential and Integral Equations Volume 14, Number 12, December 2001, Pages 1511–1530

ON THE FUČIK SPECTRUM WITH INDEFINITE WEIGHTS

M. Alif and J.-P. Gossez
Dép. Mathématique, C.P. 214
Université Libre de Bruxelles, 1050 Bruxelles, Belgium

(Submitted by: Klaus Schmitt)

1. Introduction

This paper is partly concerned with the one-dimensional asymmetric problem with weight

$$\begin{cases} Lu = m(t)(au^+ - bu^-) \text{ in } (T_1, T_2), \\ u(T_1) = u(T_2) = 0. \end{cases} \tag{1.1}$$

Here $Lu := -(p(t)u')' + q(t)u$, p, q and $m \in C[T_1, T_2]$, $p(t) > 0$ on $[T_1, T_2]$, $q(t) \geq 0$ on $[T_1, T_2]$, $m(t) \not\equiv 0$ and $u^{\pm} := \max\{\pm u, 0\}$. The associated Fučik spectrum is defined as the set Σ of those $(a, b) \in \mathbb{R}^2$ such that (1.1) has a nontrivial solution u.

The description of this spectrum Σ is classical and explicit when $Lu = -u''$ and there is no weight; i.e., $m(t) \equiv 1$ (cf. [6], [9]). The same general picture for Σ remains valid when L is as above and $m(t) > 0$ on (T_1, T_2) (cf. [8], [5], [12]) : Σ is made of the two lines $\mathbb{R} \times \lambda_1^m$ and $\lambda_1^m \times \mathbb{R}$ together with a sequence of hyperbolic-like curves in $\mathbb{R}^+ \times \mathbb{R}^+$ passing through $(\lambda_k^m, \lambda_k^m)$, $k \geq 2$; one or two such curves emanate from each $(\lambda_k^m, \lambda_k^m)$, and the corresponding solutions of (1.1) along these curves have exactly $k - 1$ zeros in (T_1, T_2). Here $(0 <)\lambda_1^m < \lambda_2^m < \cdots \to +\infty$ denotes the sequence of eigenvalues of the associated linear problem

$$\begin{cases} Lu = \lambda m(t)u \text{ in } (T_1, T_2), \\ u(T_1) = u(T_2) = 0. \end{cases} \tag{1.2}$$

One of our purposes in this paper is to investigate the situation where the weight function $m(t)$ in (1.1) *changes sign* in (T_1, T_2). In that case it is well-known that the eigenvalues in (1.2) form a double sequence : $-\infty \leftarrow$

Accepted for publication: September 2000
AMS Subject Classifications: 34B15, 34L05.