Differential and Integral Equations Volume 14, Number 12, December 2001, Pages 1487–1510

PARABOLIC PROBLEMS WITH NONLINEAR DYNAMICAL BOUNDARY CONDITIONS AND SINGULAR INITIAL DATA

José M. Arrieta[1]
Departamento de Matemática Aplicada, Facultad de Matemáticas
Universidad Complutense de Madrid, 28040 Madrid, Spain

Pavol Quittner[2]
Institute of Applied Mathematics, Comenius University
Mlynská dolina, 84248 Bratislava, Slovakia

Aníbal Rodríguez-Bernal[1]
Departamento de Matemática Aplicada, Facultad de Matemáticas
Universidad Complutense de Madrid, 28040 Madrid, Spain

(Submitted by: Herbert Amann)

1. Introduction

In this paper we consider the following parabolic problem with dynamic boundary conditions:

$$\left.\begin{array}{rcll} u_t + \mathcal{A}u &=& f(x,t,u,\nabla u), & x \in \Omega,\ t>0, \\ (\gamma u)_t + \mathcal{B}u &=& g(x,t,\gamma u), & x \in \Gamma,\ t>0, \\ u(x,0) &=& u_0(x), & x \in \Omega, \\ (\gamma u)(x,0) &=& v_0(x), & x \in \Gamma, \end{array}\right\} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ of class C^2, $\Gamma = \partial\Omega$, ν denotes the outer normal on Γ, γ is the trace operator, and $\mathcal{A}u = -\Delta u + \omega u$, $\mathcal{B}u = u_\nu + \omega u$. Although we will consider this particular case, the techniques we use can also be applied to the case of systems in which, as in [10], $\mathcal{A}u = -\partial_j(a_{jk}\partial_k u) + a_j\partial_j u + a_0$, $\mathcal{B}u = a_{jk}\nu^j\gamma\partial_k u + b_0\gamma u$, with smooth-enough coefficients. On the nonlinear terms, f and g, we assume that they are smooth functions with

[1]Partially supported by DGES PB96-0648.
[2]Partially supported by Swiss National Science Foundation and VEGA Grant 1/7677/20.
Accepted for publication: September 2000
AMS Subject Classifications: 35K60.