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ON THE STOCHASTIC KORTEWEG-DEVRIES EQUATION DRIVEN BY WHITE NOISE

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Introduction. We investigate several properties of certain solutions of the stochastic Korteweg-deVries equation

$$U_t - 6UU_x + U_{xxx} = \xi(t) \tag{1}$$

where $\xi(t) = \xi(t, \omega)$ is a random process, with $\omega \in \Omega, (\Omega, B, P)$ a probability space. Basic to our approach is a relation discovered by Miura between solutions of (1) and the usual Korteweg-deVries equation

$$u_t - 6uu_x + u_{xxx} = 0. \tag{2}$$

Miura [1] observed that if $u(x,t)$ satisfies (2), and we define

$$U(x,t,\omega) = u(x + 6\int_0^t W(s,\omega)ds, t) + W(t,\omega), \tag{3}$$

where

$$W(t,\omega) = \int_0^t \xi(s,\omega)ds, \tag{4}$$

then $U(s,t,\omega)$ satisfies (1). Conversely, if $U(x,t,\omega)$ satisfies (1), and we define

$$u(x,t,\omega) = U(x - 6\int_0^t W(s,\omega)ds, t, \omega) - W(t,\omega), \tag{5}$$

then, for every $\omega \in \Omega, u(x,t,\omega)$ satisfies (2).

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