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CONTROLLABILITY OF THE HEAT EQUATION WITH MEMORY

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1. Introduction. This work is concerned with the controllability of the equation

$$\begin{cases} y_t(x,t) - \gamma\triangle y(x,t) - \displaystyle\int_0^t a(t-s)\triangle y(x,s)\,ds = m(x)u(x,t) \text{ in } Q \\ y(x,0) = y_0(x) \quad \text{in } \Omega \\ y(x,t) = 0, \quad \text{in } \Sigma, \end{cases} \tag{1.1}$$

where γ is a nonnegative constant, Ω is an open bounded subset of R^n with a sufficiently smooth boundary $\partial\Omega$ (of class C^2 for instance), $Q = \Omega \times (0,T)$, $\Sigma = \partial\Omega \times (0,T)$ and $m(\cdot)$ is the characteristic function of an open subset ω of Ω. Finally, $a \in C^\infty(0,+\infty)$ is a locally integrable *completely monotone kernel*; i.e.,

$$(-1)^j a^{(j)}(t) \geq 0 \quad \forall t > 0, \quad j = 0,1,\ldots. \tag{1.2}$$

This equation is relevant in heat conduction in materials with memory (see e.g. [4], [10]). We recall that if $\gamma > 0$, then for each $y_0 \in H_0^1(\Omega)$ and $u \in L^2(Q)$, the equation (1.1) has a unique solution $y \in W^{1,2}([0,T];L^2(\Omega)) \cap L^2(0,T;H_0^1(\Omega)\cap H^2(\Omega))$. If $y_0 \in L^2(\Omega)$ then (1.1) has a unique solution $y \in C([0,T];H_0(\Omega))$ such that $y \in W^{1,2}([\delta,T];L^2(\Omega)) \cap L^2(\delta,T;H_0^1(\Omega)\cap H^2(\Omega))$ for each $0 < \delta < T$. Denote by y^u the solution to (1.1).

The control system (1.1) is said to be *approximately controllable* if for each $y_0 \in L^2(\Omega)$ and all $y_1 \in L^2(\Omega)$ there is $\{u_\varepsilon\} \subset L^2(Q)$ such that for $\varepsilon \to 0$ $y^{u_\varepsilon}(T) \longrightarrow y_1$ strongly in $L^2(\Omega)$.

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