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INTEGRAL METHODS FOR THE WAVE OPERATOR ON LIPSCHITZ CYLINDERS

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1. Introduction

Integral equation methods have been used lately with tremendous success in the treatment of a large variety of elliptic and even parabolic boundary problems in nonsmooth domains. See, e.g., the survey [4] and the references therein. The essence of this approach is to choose a suitable candidate, resembling a formal convolution with the fundamental solution of the differential operator at hand, and then to reduce the original problem to a certain boundary integral equation. This, in turn, involves operators whose nature is, by now, properly understood, thanks to major advances in harmonic analysis in the last two decades.

The extent to which a similar approach works for hyperbolic PDE's is far less clear. Indeed, there are essential differences between the singular set of the fundamental solutions for hyperbolic PDO's, on the one hand, and elliptic or parabolic PDO's, on the other hand.

In this note we develop a different line of attack. The starting point is the classical idea of using the Laplace transform and reducing the original hyperbolic boundary value problem to a family of elliptic boundary value problems, indexed by ω, the Laplace variable. Returning to the original setting (via the inverse Laplace transform) requires solving this family of elliptic PDE's with estimates in which all relevant constants depend *explicitly* on the parameter ω. Establishing such estimates, which is the crux of the matter, is a delicate step due to the subtle dependence of the solution of the problem at hand on ω. Among other things, we need to consider appropriate norms and spaces so that the integral equation method works. The main

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