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INTEGRODIFFERENTIAL EQUATIONS WITH PARAMETER-DEPENDENT OPERATORS

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1. Introduction

Let X be a Banach space with norm $\|\cdot\|$. Let P be an open subset of a finite-dimensional normed linear space $\mathcal{P}$ with norm $|\cdot|$. Consider the abstract integrodifferential equation

$$u'(t) = A(\varepsilon)u(t) + \int_0^t \big(F(t-s)A(\varepsilon) + K(t-s) \big) u(s)\, ds + f(t), \quad t \geq 0,$$
$$u(0) = u_0, \tag{1.1}$$

where ε is a multiparameter. For each $\varepsilon \in P$, $A(\varepsilon)$ is a closed non-densely defined linear operator and $F(t)$ and $K(t)$ are bounded operators on a Banach space X with $\|F(t)\| \leq M_1$, $\|F'(t)\| \leq M_2$, $\|K(t)\| \leq M_3$, $t \geq 0$.

One way to consider this problem is via an ODE reformulation of (1.1) determined by examining the "flow":

$$\begin{bmatrix} u(0) \\ f(\cdot) \end{bmatrix} \longrightarrow \begin{bmatrix} u(t) \\ \int_0^t F(t-\tau+\cdot)A(\varepsilon)u(\tau)\, d\tau + f(t+\cdot) \end{bmatrix};$$

equation (1.1) can then be replaced by an ordinary differential equation in a product space of the following form:

$$\begin{bmatrix} u(t) \\ v(t) \end{bmatrix}' = \begin{bmatrix} A(\varepsilon) & \delta \\ F(\cdot)A(\varepsilon) + K(\cdot) & D_s \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} f(t) \\ 0 \end{bmatrix}, \quad \begin{bmatrix} u(0) \\ v(0) \end{bmatrix} = \begin{bmatrix} u_0 \\ 0 \end{bmatrix} \tag{1.2}$$

on $\mathcal{Z} = X \times \mathcal{C}$ with the usual norm, where $\mathcal{C}$ is the space of bounded uniformly continuous functions on $[0, \infty)$ into X with the usual sup norm

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