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BIFURCATION FROM INFINITY IN A CLASS OF NONLOCAL ELLIPTIC PROBLEMS

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1. Introduction

We study the nonlocal problem

$$-\Delta u = \frac{\mu f(u)}{\left[\int_\Omega f(u)dx\right]^p} \text{ in } \Omega,\ u > 0 \text{ in } \Omega,\ \ u = 0 \text{ on } \partial\Omega, \tag{1.1}$$

where Ω is a bounded domain in R^N $(N \geq 1)$ with C^2 boundary $\partial\Omega$, p is a constant, μ is a bifurcation parameter, and f is a locally Lipschitz continuous function on $[0, \infty)$ and satisfies

$$f(u) > 0 \text{ for } u \geq 0,\ \overline{\lim}_{u\to\infty} \frac{f(u)}{u} = 0. \tag{1.2}$$

Such problems arise in various situations of practical importance, such as modelling Ohmic heating and plasma physics (see, e.g., [18, 19, 4, 13]), and have attracted considerable attention in recent years; we refer to [3, 4, 13, 14, 18, 19, 20], and the references therein for more details.

Condition (1.2) shows that $f(u)$ is sublinear near infinity. It is well-known that for the corresponding local problem

$$-\Delta u = \lambda f(u) \text{ in } \Omega,\ u > 0 \text{ in } \Omega,\ u = 0 \text{ on } \partial\Omega, \tag{1.3}$$

bifurcation from infinity at a finite λ value is possible only if the nonlinearity $f(u)$ is asymptotically linear or superlinear near infinity. Under condition (1.2), this is not possible; instead, bifurcation from infinity occurs exactly at $\lambda = \infty$, i.e., there exists a sequence of solutions (λ_n, u_n) of (1.3) such that $\lambda_n \to \infty$, $\|u_n\|_\infty \to \infty$; moreover, if (λ'_n, u'_n) is an arbitrary sequence of solutions to (1.3) with $\|u'_n\|_\infty \to \infty$, then we must have $\lambda'_n \to \infty$.

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