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NONTRIVIAL SOLUTIONS OF MOUNTAIN PASS TYPE OF QUASILINEAR EQUATIONS WITH SLOWLY GROWING PRINCIPAL PARTS

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1. Introduction

In this paper, we study the existence of nontrivial solutions of quasilinear elliptic equations of the form

$$-\mathrm{div}(a(|\nabla u|)\nabla u) = g(x,u) \text{ in } \Omega \tag{1.1}$$

with boundary condition

$$u = 0 \text{ on } \partial\Omega, \tag{1.2}$$

in the case where the function $a(t)t$ has very slow growth. Here, $g(x,u)$ is the lower order term and the function

$$\phi : t \mapsto a(t)t, \ t \in \mathbf{R},$$

represents the principal (higher order) part of the equation. We assume that ϕ is an increasing, continuous, odd function vanishing at 0 and put

$$\Phi(t) = \int_0^t \phi(s)ds \ (t \in \mathbf{R}).$$

The classical case $\Phi(t) = t^2$ corresponds to the semilinear Laplace equation. When $\Phi(t) = t^p$ $(p > 1)$, we have what is called a p-Laplacian equation. A growing literature is devoted to this case. The next natural step is to study (1.1)–(1.2) in the case where Φ is a Young function. This is the problem we are interested in here. If $g(x,0) = 0$ then 0 is always a trivial solution of (1.1)–(1.2). We are here with the existence of nontrivial ones. For this purpose, we shall use a version of the Mountain Pass Theorem. However, we are interested here in the case where Φ is growing very slowly, that is, $\Phi(t) = o(t^p)$ as $t \to \infty$ for all $p > 1$. Three issues arise:

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